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BOOK OF ABSTRACTS

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Dynamics of the spintransfer nanooscillator under spin wave impact induced by external signal

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In current work we research dynamics and bifurcations of the model of spin transfer nanooscillator, driven by external signal exiting spin waves in ferromagnetic layer of nanooscillator. We studied parameter space of coupling strength and frequency detuning for division into subareas corresponding to topologically different dynamical regimes. It is shown that frequency locking band is not a monotonic function of coupling strength (in contrast to classical case). Scenarios of frequency locking loss may differ depending on the value of coupling strength. Oscillations suppression is demonstrated as the consequence of coupling strength increment.

Soliton solutions of vector defocusing Gross-Pitaevskii equation: a method for comprehensive description?

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The Gross-Pitaevskii equation (GPE) is one of the basic equations to describe the phenomenon of Bose–Einstein condensation (BEC) in so-called mean-field approximation. In this case $\Psi(t,x)$ describes the macroscopic wave function of the condensate. The function $\Psi(t,x)$ may be scalar, in the case of single-component condensate, or vector if the condensate consists of multiple species. For binary condensate, the governing vector GPE describes the dynamics of two scalar components $\Psi_1(x,t)$ and $\Psi_2(x,t)$,

$$\begin{cases} i\Psi_{1,t} = -\Psi_{1,xx} + V(x)\Psi_1 + (|\Psi_1|^2 + \beta|\Psi_2|^2)\Psi_1, \\ i\Psi_{2,t} = -\Psi_{2,xx} + V(x)\Psi_2 + (\beta|\Psi_1|^2 + |\Psi_2|^2)\Psi_2. \end{cases} \quad (1)$$

Here $V(x)$ is a trap potential and $\beta > 0$ is responsible for interspecies relations. An important class of solution of GPE are stationary solitonic modes defined as $\Psi_{1,2}(t,x) = e^{i\omega_{1,2}t}\psi_{1,2}(x)$, $\psi_{1,2}(x) \to 0$ as

$x \to \pm\infty$. The system for real $\psi_{1,2}(x)$ reads

$$\begin{cases} \psi_{1,xx} + (\omega_1 - V(x))\psi_1 - (\psi_1^2 + \beta\psi_2^2)\psi_1 = 0, \\ \psi_{2,xx} + (\omega_2 - V(x))\psi_2 - (\beta\psi_1^2 + \psi_2^2)\psi_2 = 0. \end{cases} \quad (2)$$

The aim of this study is to present the most complete description of solitons in this model. The idea of the approach is to make use of the fact that the "most part" of the solutions of Cauchy problem for (2) are *singular*, i.e., they tend to infinity at some finite point of x axis. Since the solutions of (2) that vanish at $+\infty$ form 2D manifold W_s, one can select two parameters C_1 and C_2 to fix a solution $(\psi_1(x), \psi_2(x))$ at W_s, and put in correspondence to the pair (C_1, C_2) the value $x = x(C_1, C_2)$ where the solution collapses being continued from $x = +\infty$ limit. We argue that properly arranged numerical study of the function $x = x(C_1, C_2)$ allows to give comprehensive information about possible solitonic modes coexisting for given potential $V(x)$ and parameters β and $\omega_{1,2}$.

Coherence-incoherence transition in an ensemble of coupled maps

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We present numerical results for the bifurcation transition from coherence to incoherence in an ensemble of nonlocally coupled chaotic maps. It is firstly shown that two types of chimera states, namely, amplitude and phase, can be found in a network of coupled logistic maps. The effect of intermittency between the amplitude and phase chimera states is revealed. The analysis of mutual spatial correlation functions is conducted for the indicated types of chimera states and their peculiarities are discussed. We reveal a bifurcation mechanism by analyzing the evolution of snapshots and space-time profiles with varying coupling coefficient and formulate the necessary and sufficient conditions for realizing the chimera states in the ensemble.

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Stochastic Modeling of Slow Granular Flows

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Granular media exhibit complex behavior as they undergo phase transition from elasto-plastic to viscoelastic regime. We develop a stochastic model to capture the coupling of local plastic deformations and nonlocal stress waves along force chains in dense slow deformations. A combination of fluctuations in the velocity field and a stochastic differential equation describing force waves along connected particles is used. The model explains the nonlocal Helmholtz-like behavior of fluidity recently predicted in the literature. Finally, we study the stability of large pores in a granular medium under compaction to show the presence of bifurcations depending on pore size and distribution.

Bifurcations of periodic solutions of a generalized Hill problem

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We consider a generalization of the well-known celestial mechanics problem – Hill problem. This generalization allows to connect families of periodic solutions of the Kepler problem in uniformly rotating frame with corresponding families of the Hill problem. Bifurcations of the libration points and main families of periodic orbits of the Kepler problem are described.

Hamiltonian of the generalized Hill problem (briefly GHP) is

$$H = \frac{1}{2}\left(y_1^2 + y_2^2\right) + x_2 y_1 - x_1 y_2 - \frac{1}{|\mathbf{x}|} + \varepsilon\left(-x_1^2 + \frac{1}{2}x_2^2\right),$$

where $\mathbf{x}$ and $\mathbf{y}$ are vectors of coordinates and momenta correspondingly and parameter $\varepsilon \in [0; 1]$. For $\varepsilon = 0$ one gets the Kepler problem in uniformly rotating frame (sinodical Kepler problem or SKP shortly), which is an integrable one. For $\varepsilon = 1$ one gets the Hill problem, which is an nonintegrable.

Bifurcations of families of symmetric periodic solutions of the SKP are considered under the Hill problem bifurcation with the help of the normal forms methods. It is shown that the union of the family Id of direct circular orbits and the countable set of families of doubly symmetric elliptic orbits E_N, $N = (p + 1)/p$ or $N = (p + 1)/(p + 2)$, where p is an odd positive number, forms a common family, which characteristic is a kind of labyrinth curves in coordinates a – big semiaxis and e – eccentricity of elliptic orbit. We denote these families with IdE^+ and IdE^- correspondingly. Some parts of the curve IdE_p^+, where $p = 2k - 1$, were numerically continued into the corresponding families of doubly symmetric periodic solutions of the Hill problem.

Numerical investigation shows that for $p > 1$ the part IdE_p^+ can be continued into the doubly periodic family of the Hill problem. Such family contains orbits which make flyby the libration points L_1 and L_2 in a such order: $k - 1$ revolutions around the L_1 and then $k - 1$ revolutions around the L_2.

A simulation model of population dynamics taking into account nonlocal interactions

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The paper proposed a computer model describing the spatio-temporal dynamics of populations of interacting with a renewable resource. Described in detail the life cycle of individuals. The proposed algorithm for spatial movement of individuals within the area, taking into account nutritional and social activity. Model population consists of a set of individuals. Each individual is characterized by its mass, which we regard as a kind of "energy" which is spent in various physiological processes such as movement, reproduction and others. The model takes into consideration basal metabolism, the energy cost of movement, birth and feeding the offspring is taken into account the change in "energy efficiency" functioning in aging individuals. The main purpose of the simulation is to study the impact of different ecological and physiological parameters describing the individual, the formation of various spatio-temporal modes of behavior of the population. Of particular interest is the modeling gregarious and territorial animal behavior and the influence of the mobility of individuals. The described computational experiments with the model, which simulate the movement of animals within the area, and describes a simulation experiment when group type of animal behavior due to changes in the characteristics of the environment becomes an individual, then due to changes in environmental parameters and animal behavior formed the herd in the future goes back to gang-type behavior.

Stabilization of hyperbolic Plykin attractor by the Pyragas method

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In the present contribution consideration is being given to an autonomous physical system which is characterized by the presence of the attractor of a hyperbolic type. We study the possibility of controlling and stabilizing the Plykin attractor of this type by the Pyragas method. The choice of the method of control. As such it is possible to use an external signal or the introduction of additional delayed feedback. (Both methods can be realized primarily during the schematic simulation then in a real experiment). It might also be interesting to think about the realization of a more complicated scheme of control of the type suggested in the work for the stabilization of unstable periodic orbits belonging to the attractor.

Shilnikov bifurcation in certain systems

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In this talk we present several systems from applications for which it is proven the existence of Shilnikov bifurcations of different homoclinic orbits (HO). Namely, Shilnikov-Gavrilov-Gonchenko bifurcations of HO of periodic orbits; Shilnikov-Afraimovich-Bykov bifurcations of HO in Lorenz type systems; Shilnikov bifurcations of HO of the saddle-focus. Besides, the examples of certain systems having wild attractors are presented.

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Bifurcations and crowd dynamics on a wobbly bridge

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Time-delayed control of stability for periodic solution

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We consider the differential equation that originates in normal form for Andronov-Hopf bifurcation in case of pure imaginary eigenvalues:

$$\dot{z} = \sigma z + \gamma |z|^2 z. \tag{3}$$

Here $z(t)$ has complex values; γ and σ are given complex constants. Let represent σ as $\lambda + i\delta$. It is possible to make $\lambda = \pm\frac{1}{2}$ using normalization by time and variable z.

The system (1) has unstable periodic solution $z_*(t) = \rho_* e^{i\varphi_* t}$ where period is T_*.

$$\rho_* = \sqrt{\frac{-\lambda}{Re\gamma}}, \quad \varphi_* = \delta - \frac{\lambda Im\gamma}{Re\gamma} \quad and \quad T_* = \frac{2\pi}{\varphi_*},$$

if $\lambda \cdot Re\gamma < 0$ and $\lambda < 0$.

We investigate the ways to change stability of periodic solution using time-delayed feedback control. We research two types of the control that has one and two delays accordingly:

$$\dot{z} = \sigma z + \gamma |z|^2 z + K(z(t-T) - z). \tag{4}$$

$$\dot{z} = \sigma z + \gamma |z|^2 z + K_1(z(t-T_1) - z) + K_2(z(t-T_2) - z). \tag{5}$$

We take T, T_1 and T_2 real and positive. K, K_1 and K_2 are complex.

Our goal was to determine areas of initial system parameters (γ and σ) for which stabilization is impossible and find appropriate values of control parameters to stabilize the solution otherwise.

For the system with one delay (2) the problem was analytically solved completely and areas of γ and σ that are available for stabilization have been determined [1]. For the system with two delays (3) we analytically found D-partitions for pairs of control parameters [2]. In this case the issue of the solution stability was investigated through specially developed computer program for automatic D-partition analysis.

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Classification of constrained differential equations embedded in the theory of slow fast systems

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Formation Of Attractors In The System Of Phase Equations With Symmetry Violation

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We consider a chain of coupled phase oscillators [1] :

$$\dot{\psi}_k = \Delta_k + \varepsilon f(\psi_{k-1} + \varepsilon f(\psi_{k+1} - 2\varepsilon f(\psi_k),$$

where ψ_k is the difference of phases of neighbour oscillators and $f(\psi)$ is the coupling function. These equations are reversible and invariant relatively to the coordinate transformation $\psi_k \to \pi - \psi_{n-k}$ if the coupling function contains only odd Fourier harmonics (i.e. $f(\psi) = \sin\psi + A\sin 3\psi$) . It is known [2,3] that dynamic of such systems can be conservative relatively to the invariant manifold.

We add even harmonics to the coupling function and study the effect of symmetry breaking on the dynamics of this system. We consider $f(\psi) = \sin\psi + (A - d)\sin 3\psi + d\sin 2\psi$ as the coupling function so d is the symmetry breaking parameter.

To investigate dynamics of system we obtained numerically the Poincare map with the symmetric plane $\psi_2 = \pi/2$ as a section plane. We reveal that the stable cycles of different periods appear with the increase of the parameter d and ε. We obtain the stable and unstable manifolds of saddle cycles and show that heteroclinic structures exist.

We revealed that both regular (cycles and invariant curves) and strange attractors occur in the Poincare map plane when the symmetry violated. Also we calculated the dependencies of Lyapunov exponents of the parameter of the system and revealed that the largest Lyapunov exponent demonstrates often and sharp changes with the change of parameters which means that both bifurcations (such as the birth of resonant cycle on the invariant curve) and crises occur in this system.

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Finding first foliation tangencies in the Lorenz system

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The well-known Lorenz system is classically studied via its reduction to the one-dimensional Lorenz map, which captures the full behaviour of the dynamics of the system. The reduction requires that the stable and unstable foliations on the classic Poincaré section are transverse locally near the chaotic attractor. When this so-called foliation condition fails for the first time the Lorenz map no longer accurately represents the dynamics. We investigate the development of tangencies between the stable and unstable foliations in the Poincaré return map, the first of which marks the loss of the foliation condition. To this end, we study how the three-dimensional phase space is organised by the global invariant manifolds of saddle equilibria and saddle periodic orbits, where we make extensive use of the continuation of orbit segments formulated by a suitable two-point boundary value problem (BVP). In particular, we compute the intersection curves of the two-dimensional unstable manifold $W^u(\Gamma)$ of a periodic orbit Γ with the Poincaré section. We identify when hooks form in the Poincaré map by formulating as a BVP the point of tangency between $W^u(\Gamma)$ and the stable foliation. This approach allows us to accurately and efficiently detect additional foliation tangencies for larger values of the system parameters.

Delays can stabilise up-states in recurrently coupled neural networks

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Bistable activity of neurons is observed in numerous of experiments [1] Dynamics of single or pair of neurons with unidirectional chemical synapses were studied in previous works [2-3]. In this work we focus on studying the dynamics of ensemble of recurrently coupled bistable Hodgkin-Huxley neurons, with addition of coupling delays and Poisson noise. The neurons were distributed in a volume and had delays proportional to distance between them. Incoming noise pulses to each of neuron can evoke transitions of individual neurons from oscillatory state to steady state and vice versa so named on-off firing pattern. We found that such kind of on-off firing patterns is observed also in mean firing rate of whole network. Durations of staying in the different stable states for big time scales have exponential statistics, which is essential for bistable elements in the presence of noise, while for small times scales the statistics obey power law function. When the noise was switched off, the system moved to steady state in which partially synchronization between part of neurons is observed. In the case of absence of connection delays this phenomena cannot be observed, mean firing rate of whole network does not demonstrate two distinct states. We conclude that delays play important role in neural systems where on-off firing patterns is observed. And by using Phase Response Curve generalized for two stimuli we explain this effect.

The work is supported by The Russian Science Foundation No.14-11-00693. Calculations were performed on cluster 'Lobachevsky' of Lobachevsky University.

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Stability of some periodic orbits in a circle billiard with a small interior scatterer

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On the global minimum of the binary Weierstrass function

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In 1872, K. Weierstrass [1] introduced his function $W_{a,b}$ by a series:

$$W_{a,b}(x) = \sum_{n=0}^{\infty} a^n cos(\pi \cdot b^n x), \quad x \in \mathbf{R}.$$

He proved that this function is everywhere continuous but nowhere differentiable, under condition $0 < a < 1$, b is an odd integer, and $ab > 1 + 3\pi/2$. In 1916, G.H. Hardy [2] showed with fewer assumptions $0 < a < 1$, $ab \geq 1$ that Weierstrass function $W_{a,b}$ is nowhere differentiable and Hölder continuous of order $\log_{1/a}(b)$ everywhere.

We call *binary Weierstrass function* the function $W_a = W_{a,b}$ with $b = 2$. In this paper we study the global minimum of the function W_a for different values of a. We proved:

Theorem. 1) If $1/2 \leq a < 1$ then the global minimum of W_a on $[0; 2]$ is achieved only at the points $2/3$, $4/3$ and is equal to $1/(2a - 2)$.

2) If $0 < a < 1/8$ then the global minimum of W_a on $[0; 2]$ is achieved only at the point 1 and is equal to $(1 - 2a)/(a - 1)$.

3) If $1/8 < a < 1$ then 1 is not the point of global minimum of W_a on $[0; 2]$.

4) If $a = \Big((\sin^2(2\pi/7) + 4\sin(\pi/7)\sin(4\pi/7))^{1/2} - \sin(2\pi/7)\Big)/(4\sin(4\pi/7)) \approx 0.1887$ then the global minimum of W_a on $[0; 2]$ is achieved only at the points $6/7$, $8/7$ and is equal to $(\cos(\pi/7) - a\cos(2\pi/7) - a^2\cos(4\pi/7))/(a^3 - 1)$.

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The radar aspect of electron wave function

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Offered before author's interpretation of a scalar microparticle wave function as the projection of the signal reflected from the microparticle on the probe one, which understood as elements and operation on them in Hilbert space, is generalized to the case of the microscopic object with internal geometry and electromagnetic signals with certain circular polarization (helicity).

Magnetic dipole vs bimonopole

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Absence of direct experimental evidences is established in favor of presence at the electron of own mechanical moment, because for description of motion of his centre-of-mass in the external magnetic field of magnetic enough moment.

The classic model of electron is offered in form two coupled magnetic monopoles of opposite signs. Identification of effective brachium of this bimonopole with the classic radius of electron generates the value of magnetic charge, revealed by Dirac from the tenets of quantum mechanics.

Effect of intratrophic predation and spatial activity of zooplankton on the Turing instability in NPZ-model

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A vertically distributed three-component model of marine ecosystem is considered. State of the plankton community with nutrients is analyzed under the active movement of zooplankton in a vertical column of water. The components dynamics is described by the reaction-diffusion-advection equations. Movement of matter and organisms in depth is described by the turbulent diffusion. Directional movement of zooplankton (prey-taxis) is described by the advective terms.

The possibility of pattern formation is studied. Linear stability analysis of NPZ-model is needed to understand how the vertical mixing and search activity of zooplankton affect the biological dynamics. Analytical and numerical model studies have led to the following results. Stability of the spatially homogeneous equilibrium, the Turing instability and the oscillatory instability are examined depending on the phytoplankton uptake rate of zooplankton and density-dependent death of zooplankton. We were obtained the necessary conditions of the Turing instability in the vicinity of the spatially homogeneous equilibrium. If the consumption of phytoplankton is low the formation of spatial patterns is possible at low rates of zooplankton intratrophic predation and diffusion. With the increase in zooplankton grazing rate impact of this coefficient on the spatial instability becomes less than impact of phytoplankton diffusion and spatial movement of zooplankton.

Dynamics of modular exponent and the Higman group

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Consider the dynamical system $f : \mathbb{Z}_n \to \mathbb{Z}_n$, where $\mathbb{Z}_n = \{0, 1, \cdot, n-1\}$ is the ring of residues modulo n and f defined as

$$f(x) = Cq^x \mod n.$$

This function is supposed to be a one-way function (at least for some values of q, n) and has an extensive use in the cryptography. One may consider f as a finite dynamical systems and study the trajectories $x_{i+1} = f(x_i)$. Of course, all limit trajectories are just periodic ones. Suppose that f indeed a one-way function, then the number of "small" (relative to n) periodic trajectories should be small (relative to n). As far as I know there are no good estimation for the number of k-periodic trajectories for $k \geq 4$.

I show that there exist an "almost modular exponent" f for $q \neq 2$ such that for all $x \in \mathbb{Z}_n$ $f(f(f(f(x)))) = x$. Precisely, for any $q > 2$ for any $\epsilon > 0$ there exists N_ϵ such that for any $n > N_\epsilon$ there exists $f(x) = C(x)q^x \text{mod } n$ such that

- $f(f(f(f(x)))) = x$
- $\frac{|\{x \in \mathbb{Z}_n \mid C(x) = C(x+1)\}|}{n} \geq 1 - \epsilon.$

Probably, it is related with difficulty of estimating the number of short cycles in modular exponent. The proof of the existence of such an almost exponent based on properties of the Higman groups and does not work for $q = 2$. The talk is based on preprints:

- Harald A. Helfgott, Kate Juschenko, Soficity, short cycles and the Higman group, arXiv:1512.02135.
- Lev Glebsky, p-quotients of the G.Higman group, arXiv:1604.06359.

Variety of strange pseudohyperbolic attractors in three-dimensional generalized Hénon maps

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In this talk we focus on the problem of the existence of strange pseudohyperbolic attractors for three-dimensional diffeomorphisms. Such attractors are genuine strange attractors in that sense that each orbit in the attractor has a positive maximal Lyapunov exponents and this property is robust, i.e. it holds for all close systems. We restrict attention to the study of pseudohyperbolic attractors that contain only one fixed point. Then we show that three-dimensional maps may have only 5 different types of such attractors, which we call the discrete Lorenz, figure-8, double-figure-8, super-figure-8, and super-Lorenz attractors. We find the first four types of attractors in three-dimensional generalized Hénon maps of form $\bar{x} = y,\ \bar{y} = z,\ \bar{z} = Bx + Az + Cy + g(y, z)$, where A, B and C are parameters ($B > 0$ is the Jacobian) and $g(0, 0) = g'(0, 0) = 0$.

Strange attractors in three-dimensional nonorientable Henon maps

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We consider the problem of the existence of discrete strange homoclinic attractors (that contains only one fixed point) for three-dimensional nonorientable diffeomorphisms. We show that 6 different types of such discrete nonorientable attractors can exist: we call them a "skinny" Lorenz attractor (sLA), double Lorenz attractor (dLA), figure-8 attractor (8A), double figure-8 attractor (d8A), spiral attractor (SpirA), Shilnikov attractor (ShA). We found the attractors of four types (sLA, d8A, SpirA and ShA) in three-dimensional generalized Hénon maps of form $\bar{x} = y, \bar{y} = z, \bar{z} = Bx + Az + Cy + g(y, z)$, where A, B and C are parameters (B is the Jacobian and $B < 0$). We consider the case where $g(0, 0) = g_y(0, 0) = g_z(0, 0) = 0$ and search attractors that contain the fixed point $O(0, 0, 0)$.

Bifurcations of cubic homoclinic tangencies in two-dimensional symplectic maps

Marina Gonchenko (joint work with **Sergey Gonchenko** and **Ivan Ovsyannikov**.)

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We study bifurcations of cubic homoclinic tangencies in two-dimensional symplectic maps. The method we use is based on the construction of first return maps near a given nontransversal homoclic orbit. We distinguish two types of cubic homoclinic tangencies, and each type gives different first return maps derived to diverse conservative cubic Hénon maps. We analyse bifurcation diagrams of the cubic Hénon maps paying special attention to the problem of 1:4 resonance. In this way, we establish the structure of bifurcations of periodic orbits in two parameter general unfoldings generalizing to the conservative case the results previously obtained for the dissipative case.

Elements of contemporary theory of dynamical chaos

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The presentation consists of two lectures.

Lecture 1. Three types of dynamical chaos.

Lecture 2. Elements of modern theory of strange attractors.

In Lecture 1 we discuss new concept of dynamical chaos based on the existence of three independent form of chaos: conservative chaos, strange attractor and mixed dynamics. Two first forms of chaos are well-known, while the mixed dynamics is rather new. Formally, such classification appears by the following principle: conservative chaos – attractor and repeller coincide; strange attractor (repeller) – attractor and repeller do not intersect; mixed dynamics – attractor and repeller intersect but do not coincide. We give mathematical concept for all three forms of chaos based on the Anosov-Conley-Ruelle theory of chaos and notion of ε-orbit. Main attention will be paid to the mathematical concept of mixed dynamics. Partial topics of this theory and more concrete examples will be also presented on this conference in talks by A. Kazakov and T. Lazaro (Wednesday).

In Lecture 2 we present some elements of modern theory of strange attractors. We divide all attractors into two classes: quasiattractors and wild (pseudo)hyperbolic attractors. In particular, pseudohyperbolic attractors (or, another term, volume hyperbolic attractors) include hyperbolic and Lorenz attractors but can contain homoclinic tangencies and, hence, Newhouse wild hyperbolic sets. However, bifurcations of these homoclinic tangencies do not lead to the birth of periodic sinks (quasiattractors, conversely, allow the birth of periodic sinks, by definition). The theory of wild pseudohyperbolic attractors was laid by Turaev and Shilnikov [1,2] and we discuss some elements of this theory and give several examples of such attractors for three-dimensional Hénon-like maps. Closely related topics will be also presented in talks by K. Shinohara, I. Ovsyannikov, A. Gonchenko (Monday), A.Kazakov–I.Sataev (Tuesday) and A.Gonchenko–A.Kozlov (poster-session).

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On dynamical systems on surfaces and Anosov-Weil theory

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Roughly speaking, Anosov-Weil Theory includes the following two parts:

- a study of nonlocal asymptotic properties of simple curves on a surface by lifting these curves to an universal covering, and making a ”comparison” in a sense with lines of constant geodesic curvature;

- an application of nonlocal asymptotic properties for constructions of topological invariants for surface dynamical systems and foliations.

Especially this approaching turned up effective for dynamical systems with nontrivially recurrent motions and nontrivially recurrent invariant manifolds. The most known of such dynamical systems are pseudo-Anosov homeomorphisms, Anosov and DA diffeomorphisms and foliations with nontrivially recurrent leaves.

Our report is devoted to exposition of main aspects concerning Anosov–Weil Theory.

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Towards the global bifurcation theory on the plane with a nonsingular endomorphism of circle

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The talk provides a new perspective of the global bifurcation theory on the plane. Theory of planar bifurcations consists of three parts: local, nonlocal and global ones. It is now clear that the latter one is yet to be created. Local bifurcation theory (in what follows we will talk about the plane only) is related to transfigurations of phase portraits of differential equations near their singular points. This theory is almost completed, though recently new open problems occurred. Nonlocal theory is related to bifurcations of separatrix polygons (polycycles). Though in the last 30 years there were obtained many new results, this theory is far from being completed. Recently it was discovered that nonlocal theory contains another substantial part: a global theory. New phenomena are related with appearance of the so called sparkling saddle connections. The aim of the talk is to give an outline of the new theory and discuss numerous open problems. The main new results are: existence of an open set of structurally unstable families of planar vector fields, and of families having functional invariants (joint results with Kudryashov and Schurov). Thirty years ago Arnold stated six conjectures that outlined the future development of the global bifurcation theory in the plane. All these conjectures are now disproved. Though the theory develops in quite a different direction, this development is motivated by the Arnold's conjectures.

On semiconjugacy of the Wiliams endomorphism with a nonsingular endomorphism of circle

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It is shown that a nonsingular Williams endomorphism of a one-dimensional branched manifold with two branching points is semi-conjugate to a non-singular endomorphism of a circle of degree 3. This allows us to obtain a partial description of the non-wandering set of the Williams endomorphism. In particular, we infer from the main result that if the endomorphism is not a uniformly expanding immersion, then its nonwandering set contains an invariant Cantor set.

Adiabatic approximation of the resonance capture

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Mathematical model of capturing in resonance is reduced to some system of nonlinear ordinary differential equations with slow varying coefficients. There are different approach to analyse the problem by two scale expansions. The most famous method is adiabatic approximation. We discuss a lack of this approach in the case of small amplitude pumping. The cyclotron resonance is considered as an example from the physics.

Local dynamics of second-order equation with large delay

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Consider second-order nonlinear equation

$$\frac{d^2x}{dt^2} + \sigma\frac{dx}{dt} + x = ax(t-T) + F(x(t-T))$$

with large delay ($T \gg 1$). Here $x \in R$, $\sigma > 0$ and F is smooth nonlinear function. Let study the stability of zero equilibrium and its bifurcations.

Its stability depends of location of roots of characteristic equation

$$\lambda^2 + \sigma\lambda + 1 = a\exp(-\lambda T).$$

There are two types of critical cases. At the critical case of first type ($\sigma > \sqrt{2}$, $a = \pm 1$) infinite number of roots tend to imaginary axis when delay tends to infinity. At the second one ($\sigma < \sqrt{2}$, $a = \pm a(\sigma)$) each root has the limit in left complex half-plane, but for sufficiently large delay there are exist root λ with small real part. The number of such λ is unbounded. So, all critical cases have infinite dimension.

In critical cases the method of research based on method of normal forms. The main idea is to built special equations (we call it quasinormal forms) with help of some asymptotic substitution. These equations describes behaviour of solutions of initial problem. In the both critical cases quasinormal forms are parabolic equation without small or large parameters.

In the critical of first type, equilibrium state loss stability and periodic solutions may born during bifurcation. The number of such solutions may be any for sufficiently large T. In the critical case of second type again periodic solutions may born, but the frequency of these solutions is asymptotically large.

On the phenomenon of mixed dynamics in Pikovsky-Topaj system of coupled rotators

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A one-parameter family of time-reversible systems on 3-torus is considered. It is shown that the dynamics is not conservative, namely the attractor and repeller intersect but not coincide. We explain this as the manifestation of the so-called mixed dynamics phenomenon which corresponds to a persistent intersection of the closure of stable periodic orbits and the closure of the completely unstable periodic orbits. We search for the stable and unstable periodic orbits indirectly, by finding non-conservative saddle periodic orbits and heteroclinic connections between them. In this way, we are able to claim the existence of mixed dynamics for a large range of parameter values. We investigate local and global bifurcations that can be used for the detection of mixed dynamics.

The work was supported by Basic Research Program in HSE in 2016 (project 98).

Scenarios of transition to chaos and evolution of strange attractors in the nonholonomic model of Chaplygin top

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We consider the motion of a dynamically asymmetrical ball on a plane in the gravity field. The center of mass of the ball does not lie on any planes of inertia, and the point of contact of the ball with the plane is subject to a nonholonomic constraint which forbids slipping. Following [1] we call such a ball Chaplygin top.

Our interest in nonholonomic models is caused by the fact that (as was shown in previous studies [2,3]) such systems exhibit a wide variety of new interesting examples of strange attractors that are typical for the three-dimensional maps [4]. The aim of this study is to investigate the typical scenarios of the appearance and evolution of strange attractors in the nonholonomic model of Chaplygin top.

We show that the nonholonomic model of Chaplygin top demonstrates a comprehensive variety of scenarios of transition to chaos, in particular, torus attractors breakup in accordance with the mechanism of Afraimovich-Shilnikov [5], Feigenbaum cascade inside the synchronization domain, and via torus doubling cascade [6]. In addition, the model exhibits some typical sequences of bifurcations of regular and chaotic attractors, which include the above basic scenarios of tori destruction as their stages. Several examples of such metascenarios are discussed, one of them results in a discrete heteroclinic Shilnikov attractor [4].

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The application of modified Lagrange's and Hamilton's equations for systems with energy dissipation

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In this paper the modification of Lagrange's mechanics (making it applicable for systems with friction) is proposed. We consider the lagrangian in vector form for 3-dimensional case:

$$L = \frac{m\dot{\vec{r}}^2}{2} - u(\vec{r}, t) + \vec{f}(\vec{r}, t)\dot{\vec{r}} \quad , \tag{1}$$

where $\vec{f}$ is a special type of vector potential (and u is ordinary scalar potential). It is possible to receive the Hamilton's function by means of the standard procedure of Legendre transformation:

$$H = \frac{(\vec{R} - \vec{f})^2}{2m} + u, \tag{2}$$

where $\vec{R} = \frac{\partial L}{\partial \dot{\vec{r}}} = m\dot{\vec{r}} + \vec{f}$ is the generalized momentum. The standard Lagrange's equations are replaced with the next equations:

$$\frac{\tilde{d}}{dt}\left(\frac{\partial L(t, \vec{r}, \dot{\vec{r}})}{\partial \dot{\vec{r}}}\right) - \frac{\partial L(t, \vec{r}, \dot{\vec{r}})}{\partial \vec{r}} = 0. \tag{3}$$

The difference is that the total derivative on time is replaced with derivative of special kind (designated by the "tilde" over the letter "d"). It is expressed by the next formula:

$$\frac{\tilde{d}}{dt}(...) = \frac{d}{dt}(...) - (\vec{V}\nabla)(...), \tag{4}$$

where $\vec{V}$ is velocity. The similar changes for the Hamilton's equations are entered: the replacement of the total derivative of the generalized momentum with the derivative (4). By means of these innovations we can receive the expression

$$\frac{dE}{dt} = \vec{V}\{(\vec{V}\nabla)\vec{R}\} - \vec{V}\frac{\partial \vec{f}}{\partial t} + \frac{\partial u}{\partial t} = \vec{V}\{(\vec{V}\nabla)\vec{f}\} - \vec{V}\frac{\partial \vec{f}}{\partial t} + \frac{\partial u}{\partial t}$$

for the rate of energy change.

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Effect of optimal harvesting on the genetic diversity and dynamics of a mendelian limited population

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The purpose of work is the description and research of the simplest model situation in which the patterns of interrelated changes in the dynamics of the genetic structure and the population size are induced by interactions of evolutionary (mainly selective) and ecological (limiting population growth) factors, including the effects of harvesting on exploited populations. An example of such a model system is a diploid Mendelian panmictic population in which the genetic diversity is controlled by a single diallele locus (with alleles A and a) and ecological limitation is reduced to a decreasing dependence of adaptation on the population size and impact of harvesting - to withdrawal of part of individuals.

The designations used are the following: x_n, the population size in the n-th generation; q_n, the frequency of allele A in the n-th generation (hence, $(1-q_n)$ is the frequency of allele a); $W_{AA}(n)$, $W_{Aa}(n)$ and $W_{aa}(n)$ the fitnesses of the genotypes AA, Aa and aa, respectively, in the n-th generation, u - proportion of withdrawal . The system of recurrence equations used to describe the changes in the size and genetic structure of the population is as follows:

$$\begin{cases} x_{n+1} = \overline{W}_n(x_n)x_n(1-u), \\ q_{n+1} = q_n\left(W_{AA}(x_n)q_n + W_{Aa}(x_n)(1-q_n)\right)/\overline{W}_n(x_n), \end{cases}$$

where $\overline{W}_n(x_n) = W_{AA}(x_n)q_n^2 + 2W_{Aa}(x_n)q_n(1-q_n) + W_{aa}(x_n)(1-q_n)^2$ is the mean fitness of the population of the n th generation. We hypothesize that fitnesses linearly depend on the population size $W_{ij} = 1 + R_{ij} - \frac{R_{ij}}{K_{ij}}x$.

Each genotype is characterized by its resource (K_{ij}) and Malthusian (R_{ij}) parameters. To simplify mathematical manipulations, let us additionally assume that all genotypes have the same fitness at a certain population size (x^*).

Equilibrium values of population size and frequency frequency of allele the models providing the maximum volume of withdrawal are found.

It is shown that the linear form of the fitness function and described relations between the parameters of stationary model genetic makeup does not depend on the number of stationary values. It is shown that the conditions for the existence of equilibrium values generally in the absence of harvesting and its effects are the same.

Numerical study of the impact of harvesting with the constant of withdrawal on the dynamics of populations showed that the at a harvesting with an any optimal withdrawal proportion leads to a stabilization of the population size and the frequency of allele A. In addition, it is shown that the harvesting with an optimal withdrawal proportion the population size remains at the equilibrium level at any values of model parameters, but harvesting can lead to changes in genetic composition case any of the optimal proportion of withdrawal translates equilibrium population size through x^*. Thus, harvesting can cause changes to the selection results and cause destruction or contribute to the maintenance of polymorphism.

Therefore, the optimal harvesting may change the conditions of natural selection and not only result in changes in the dynamics of the exploited population, but also change the direction of genetic evolution.

Inertial manifolds for reaction-diffusion-advection problems in a ring of nonlocally coupled logistic maps

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Transformations of cluster basins and phase space in a ring of nonlocally coupled logistic maps

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Coupled map lattice are actively used in various sciences. For example these are systems of coupled neurons, populations of animals, people groups of different status or living in different territories. In this sense coupling is a flow of substances, energy, individuals, peoples or information which change the state of both coupled elements. In this vein, the nonlocal coupling corresponds to long-range relations between oscillating elements. Increasing the coupling radius is expressed as transmission of substance, energy and information to larger and larger distances and smoothing the differences between distant elements. In this context of particular interest are the phenomena of long-range interactions. In particular the patterns of clustering or cluster destruction which lead to chaotic synchronization or turbulent dynamics may provide a fundamentally new explanation of the people groups formation united by common features (eg, national, cultural or political). In this case it is important to consider not only coupled strength, coupled radius and parameters of local dynamic but also the initial states of the single elements (oscillators).

In this report, for discussion the results of the study of conditions leading to the formation and destruction of cluster in the ring of nonlocal coupled maps are submitted. We consider two types of unimodal map – the logistic and Ricker population map. The structure of phase and the parameter space is studied at various coupling radius and values of the control parameters. It was found their design is similar to multistable mode domains of systems coupled maps of smaller dimension (dimension is equal to the number of clusters). It is established with increasing coupling radius the cluster domain size of existence and stability is reduced and closer to domains of smaller dimension system. It is shown phase space consists of embedded in each attraction basins of clusters which differ in size and location on the ring. As a result, when we fixing the some initial state by only one kind of clusters structure (size, location and values of each variable) it seem the existence regions of such clusters structure are much smaller than it actually. Therefore study was conducted on different structure, shapes, location, sizes and number of clusters. It was found three- and four- cluster states are realized in a wider range of parameters than the two-cluster state. Analogously the domain of state with one small and one large cluster is greater than the domain of the two equal clusters.

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Clustering in the linear chain and ring of Ricker population models at the colonization of the linear area

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This study is devoted the study of mechanisms and pattern formation of spatial dynamics in populations which are chains of local population groups or subpopulations coupled by migration. We study the movement of individuals in a linear area from a single and not initially empty subpopulation. Linear form of habitat occurs for animal species whose habitat stretches along of some natural object (eg. the river bed, valley complex, the mountainside, the boundaries between forest and meadow). The settlement of the area from a single local population or colonization is observed in the case of the catastrophic destruction of the population by almost the whole area and its further recovery from the only surviving population. Besides, this situation is typical for new and pioneer species colonizing area.

As a model of this population we use the linear chain or ring of locally dissipative coupled Ricker population model. It is shown the colonization of the linear area occurs in two stages. At the first one there is an initial filling of area which is accompanied by the rapid growth of population, a series of relaxation oscillations and the formation of a quasi-stationary dynamics when a subpopulations for a long time hardly fluctuates. At the second one quasi-stationary dynamics collapses and stable population fluctuations are formed. On a space-time plots (number of oscillator and discrete time) these stages look like two triangles (first – the zero population number, second – a constant, further follow the stripes of minimum and maximum number). Under certain lengths of chain or ring the fluctuations in different parts of the area are asynchronous and are formed clusters. In some cases subpopulation boundary separating these clusters does not fluctuate. The latter is defined by parity or odd of number of subpopulations in the chain or ring.

Unexpectedly the second stage of colonization completely missing at a very low coupling strength (migration). In this case the initial filling of the area passes for much longer. Immediately thereafter the periodic dynamics is formed which for various lengths of chains leads to clusters of different sizes and structures. Thus there is the size and structure of the clusters dependence on chain or ring length of population coupled by migration.

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Hyperbolic chaos in mechanical and electronic systems

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Mixed dynamics in planar reversible maps with a symmetric couple of quadratic tangencies

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Stable dipole solitons and soliton complexes in the nonlinear Schrödinger equation with periodically modulated nonlinearity

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Nonlinear lattices (NLs) arise nowadays in many physical applications, such as nonlinear optics and models of Bose-Einstein condensates. In the latter case, NLs appear due to periodic modulation of the local nonlinearity strength by means of properly patterned magnetic or optical field. This setting is modeled by the equation of the nonlinear Schrödinger type with a periodically modulated coefficient in front of cubic term:

$$i\Psi_t + \Psi_{xx} + U(x)\Psi + P(x)|\Psi|^2\Psi = 0.$$

It is known that this model supports simplest (single-peaked, spatially symmetric) solitons. These objects are stable in some interval of the respective chemical potential.

In our study we address the following two issues: (i) do there exist more complex solitons in this model, and (ii) if yes, which of them are stable? Addressing the former issue, we have found that the model supports a plethora of complex localized modes, that can be coded by means of bi-infinite words of alphabet with an infinite number of symbols. Addressing the latter issue, we have found that a majority of complex nonlinear modes are unstable. However, there are two stable species: (a) single-peaked fundamental solitons, and (b) a new species of dipole solitons, namely, narrow spatially antisymmetric modes, which are squeezed, essentially, into a single NL cell. The stability of these modes is predicted, in a certain region of values of the chemical potential, by a variational approach, and has been checked by means of the linear stability analysis, as well as by direct numerical simulations.

When rare events dominate transport: stickiness in Hamiltonian systems

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In this talk I will discuss the problems related to anomalous transport and stickiness in low dimensional Hamiltonian systems. We firts introduce an exponent to characterize transport based on convergence of finite time observable average distributions. Using tools inspired from this approach we shall study the stickiness of regular island, and exhibit a potential problem with Kac Lemma. This problem actually implies that the ergodic measure is singular, leaving most of physically relevant initial conditions associated to the invariant lebesgue measure irrelevant: rare becomes important, but this could be as well a numerical artefact.

On dynamics of diffeomorphisms of 3-manifold with one-dimensional surface basic sets

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Let G be a class of diffeomorphisms of 3-manifold, such that each diffeomorphism from this class is a locally direct product of a DA-diffeomorphism of 2-torus and rough diffeomorphism of the circle. In this report we study dynamics of diffeomorphisms from the class G. It is proved that the class of topological conjugacy of such diffeomorphism is completely determined by combinatorial invariants, namely hyperbolic automorphism of the torus, a subset of its periodic orbits, the number of periodic orbits and the serial number of the diffeomorphism of the circle.

Heterodimensional cycles born at bifurcations of Shilnikov loops

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Sectional Anosov flows: Existence of Venice masks with two singularities

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A *sectional-Anosov flow* on a manifold M is a C^1 vector field inwardly transverse to the boundary for which the maximal invariant is sectional-hyperbolic [3]. We say that a sectional Anosov flow is a *Venice mask* if it has dense periodic orbits which is not transitive [2], [5], [4], [1].

The only known examples of venice masks have one or three singularities, and they are characterized by having two properties: are the union non disjoint of two homoclinic classes and the intersection of its homoclinic classes is the closure of the unstable manifold of a singularity.

We provide two examples of venice masks with two singularities (with different approaches). Here, each one is the union of two different homoclinic classes. However, for the first, the intersection of homoclinic classes is the closure of the unstable manifold of two singularities. Whereas for the second, the intersection of homoclinic classes is just a hyperbolic periodic orbit.

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An identical period-doubling route to chaos in a family of 3-D sinusoid discrete maps

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In this paper, we have proposed a new family of sinusoid discrete chaotic map with two parameters, this family can exhibits a chaotic attractors form same typical period-doubling bifurcation route to chaos. This physical phenomenon is justified by numerical investigation.

Contractibility of manifolds by means of stochastic flows

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In [Probab. Theory Relat. Fields, **100** (1994) 417–428] Xue-Mei Li studied stability of stochastic differential equations and the interplay between the moment stability of a SDE and the topology of the underlying manifold. In particular, she gave sufficient condition on SDE on a manifold M under which the fundamental group $\pi_1 M = 0$. We prove that in fact under essentially weaker conditions the manifold M is contractible, that is all homotopy groups $\pi_k M$, $k \geq 1$, vanish.

Let $\mathcal{T} = (\Omega, \mathcal{F}, \mathbf{P})$ be a probability space, and $\mathcal{B}(X)$ denotes the Borel algebra of subsets of a topological space X. A map $\xi : M \times [0, +\infty) \times \Omega \to M$ wil be called a *stochastic deformation* whenever ξ is $\mathcal{B}(M \times [0, +\infty)) \times \mathcal{F} - \mathcal{B}(M)$-measurable and has the following property: there exists

$N \in \mathcal{F}$ of measure 0 such that for each $\omega \in \Omega \setminus N$ the map $\xi_\omega : M \times [0, +\infty) \to M$, $\xi_t(x,t) = \xi(x,t,\omega)$, is continuous and $\xi(x, 0, \omega) = x$ for all $x \in M$.

Given a stochastic deformation ξ one can define the following σ-additive probability measures $\mu_{x,t}$, $(x,t) \in M \times [0, +\infty)$ on M: $\mu_{x,t}(A) := \mathbf{P}\{\omega \in \Omega : \xi(x,t,\omega) \in A\}$.

Theorem. *Suppose ρ is a complete Riemannian metric on M and $\xi : M \times [0, +\infty) \times \Omega \to M$ is a stochastic deformation having the following properties:*

(i) *the map $\xi_{t,\omega} : M \to M$, $\xi_{t,\omega}(x) = \xi(x,t,\omega)$, is C^1 for all $t \in [0, +\infty)$ and $\omega \in \Omega \setminus N$;*

(ii) *for each compact subset $\mathbf{L}$ of the tangent bundle TM we have that*

$$\int_0^{+\infty} \sup_{(x,v)\in\mathbf{L}:\ x\in M,\ v\in T_xM} \mathbf{E}\,\|T_x\xi_{t,\omega}(v)\|dt < \infty,$$

where $\mathbf{E}f = \int_\Omega f d\mathbf{P}$ is a mean value, and the norm is taken with respect to ρ;

(iii) *there exists a point $z \in M$, a compact subset $K \subset M$, $\varepsilon > 0$ and $N > 0$ such that $\mu_{z,t}(K) > \varepsilon$ for all $t > N$.*

Then M is contractible.

High dimensional perturbations of zero entropy Lorenz maps

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We consider one-dimensional Lorenz maps and their perturbations. In [1], [2] we dealt with multidimensional perturbations of one-dimensional hyperbolic maps with positive topological entropy: $h_{top}(f) > 0$. For those maps we proved that topological entropy changes continuously. Now we study perturbations in a neighborhood of a map with zero topological entropy. This case is much more complicated, and the property of continuous dependence of $h_{top}(f)$ may fail for C^0-topology. However, if one considers Lorenz maps with zero one-sided derivatives at the discontinuity point and with C^1-topology, then $h_{top}(f)$ continuous dependence still takes place. More precisely, the result is as follows.

Theorem The function $f \to h_{top}(f)$ in the class of Lorenz maps with C^0-topology is continuous at f_0, except for the case when $h_{top}(f_0) = 0$ and the kneading invariants $K^+_{f_0}, K^-_{f_0}$ of f_0 are periodic with the same period; in the latter case, the jump of topological entropy is precisely $\frac{1}{p}\log 2$, where

p is the common period of the kneading invariants. Moreover, for the class of Lorenz maps having zero one-sided derivatives at the discontinuity point and with C^1-topology, such an exceptional case is impossible, and thus, the topological entropy depends continously on the map.

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Non-trivial attractors for Anosov diffeomorphisms.

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We show that there is a C^1 smooth Anosov diffeomorphism on a two-torus such that the omega-limit set of almost any point with respect to the Lebesgue measure does not intersect some fixed open set. Therefore, the Milnor attractor (the smallest closed set that contains the omega-limit sets for almost every point) of this Anosov diffeomorphism does not coincide with the whole phase space. In fact, in our example it has zero measure. This result shows the drastic difference between the C^1 and C^2 Anosov diffeomorphisms. For C^2 smooth transitive Anosov diffeomorphisms, the classical reuslts om SRB-measure imply that the Milnor attractor coincide with the whole phase space.

A.D.Morozov, K.E.Morozov On transitory shift in pendulum type equations

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The two-dimensional non-autonomous equations of pendular type are considered: the Josephson equation and the equation that describes oscillations of a body suspended on ropes. It is supposed that these equations are transitory, i.e. non-autonomous only on a finite time interval. In the conservative case the measure of transport from oscillations to rotations is established. For the non-conservative case the transitory shift influence to solutions behavior is considered.

Emergence and coherence of oscillations in star and tree networks of stochastic excitable elements

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We study the emergence and coherence of stochastic oscillations in regular star and tree networks of excitable elements in which peripheral nodes receive independent random inputs. A biophysical model distal branches of sensory neuron in which nodes of Ranvier are coupled by myelinated cable segments is used along with a generic model of networked stochastic active rotators. We show that coherent oscillations can emerge due to stochastic synchronization of peripheral nodes and that the degree of coherence can be maximized by tuning the coupling strength and the size of the network.

Complex Dynamic Regimes of Age-structured Population with Density Dependent Regulation*

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Many real populations including exploited populations demonstrate complex dynamics modes, which are poorly predicted. The paper researches the model of population dynamics with age structure and density dependent regulation. We investigate and compare two situations: 1) population develops freely and 2) population is exploited. We consider the population which, by the end of each reproduction season, consists of two age groups: juveniles (immature individuals) and adults (participants of the reproduction process). To describe the population dynamics we use recurrence equations. The model of exploited population is assumed that harvesting is realized once a year after the reproductive season. The number of harvested animals is proportion to the total population size. Research of both models is done by analytical and numerical methods. It is shown there is multistability in the population model without harvesting. This phenomenon is coexistence of different dynamic regimes, namely equilibrium and 3-cycle. As a rule harvesting leads to the dynamics stabilization, however the multistability continues to exist. Irregular harvesting or non-constant harvest rate may cause of population fluctuations emergence. It is shown that even a single harvesting in the

current population size can lead to a change of the observed dynamic regime. Thus, this research showed multistability of dynamic regimes can be observed in a population system as a freely developing and exploited. There is some difficulty in predicting the dynamics of exploited population size due to the multistability. Influence of harvesting can shift the current number population from one attraction basin to another and lead to substantial changes in dynamic modes.
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Generic iterated function systems on the circle

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Joint work with V.A. Kleptsyn and Yu.G. Kudryashov.

An *iterated function system* (IFS) on a manifold M is a tuple of smooth maps $f_1, \ldots, f_s : M \to M$. One of the reasons for studying IFS's is that they (more precisely, associated step skew products over Bernoulli shift) provide a nice model example of partially hyperbolic skew products. If some interesting robust property is found for the IFS's, it is often possible to find this property for a locally generic set of diffeomorphisms (see, e.g., [1]).

Generic IFS's on the interval were studied by V. Kleptsyn and D. Volk ([2]). Among other things, they proved that the associated skew product has finitely many attractors and finitely many physical measures. However, it is unknown whether the supports of these physical measures coincide with the attractors.

Unlike IFS's on the interval, IFS's on the circle can be minimal (i.e., the $(f_1, \ldots, f_s)$-orbit of each point is dense). It turns out that this is the only difference from the interval case. Namely, for an open and dense set of IFS's on the circle such that $f_1, \ldots, f_s$ are orientation-preserving diffeomorphisms an alternative holds:

- either the IFS is minimal
- or there is an absorbing domain — a nontrivial finite union I of open intervals such that $f_i(I) \subset I$ for each $i = 1, \ldots, s$. In this case the results from [2] can be applied.

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Discrete Lorenz attractors in three-dimensional maps with homoclinic and heteroclinic tangencies

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Complex behaviour in cyclic competition bimatrix games

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Robust Chimeras in Networks

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In networks of identical oscillators a chimera state is a spatio-temporal pattern in which synchronous and asynchronous oscillations coexist. We construct stable chimeras states in coupled stars graphs. These chimeras emerge from a correlation between the isolated dynamics of the nodes and the network structure. Our starting point is a single star graph of phase oscillators with the hub node rotating faster than the leaves. When two of such star graphs are coupled one subnetwork can display coherent dynamics while the remaining subnetwork is incoherent. Our results are constructive and we can determine the macroscopic behaviour in terms of the microscopic parameters. In particular, we can construct chimeras for arbitrarily large coupling parameters.

Energy function for A-diffeomorphisms in dimensions 2 and 3

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Exponential dichotomy and bifurcation theory of the Hamiltonian operator's boundary value problems in the Hilbert space

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The report is devoted to obtaining the conditions of the solvability of weakly perturbed boundary value problems for the operator differential equation in the Hilbert space H on the interval and whole line

$$\frac{dX(t,\varepsilon)}{dt} = [A, X(t,\varepsilon)] + \Phi(t) + \varepsilon C(t)X(t,\varepsilon), t \in J \tag{6}$$

$$lX(\cdot,\varepsilon) = \alpha, \tag{7}$$

under assumption that homogeneous operator differential equation

$$\frac{dX(t)}{dt} = [A, X(t)] + \Phi(t), \tag{8}$$

admits an exponential dichotomy on the semi-axes [1,2]. Here $X(t,\varepsilon)$ is an unknown operator-valued function from the space $C^1(J, \mathcal{L}(H))$, $A \in \mathcal{L}(H)$, $[\cdot,\cdot]$ is a commutator of operators [3]

$$[A, X(t)] = AX(t) - X(t)A,$$

$C(t), \Phi(t) \in C(J, \mathcal{L}(H))$ are given strongly-continuous operator-valued functions;
$l : C^1(J, \mathcal{L}(H)) \to H_1$ is linear and bounded operator. We seek the solution of boundary value problem (6, 7) which for $\varepsilon = 0$ turns in one of the solutions of generating problem (7, 8).

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Existence and stability of bursting cycles in a neuron model with two delays

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Dynamic modes and multistability in the model of population with a simple age structure

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In this paper, we investigate dynamic modes in a population with simple age structure. We assume that by the end of each reproduction season the population consists of two age groups: juveniles and adults. The density-dependent limitation of a younger class survival regulates the population size increase. In the nature such populations are represented by small mammals, fast maturing fish, and many insects. A discrete time two-component population model can describe the dynamics of such populations. The offspring survival rate linearly depends on the number of juveniles and adults groups. The purpose of this research is to investigate evolutionary scenarios of the oscillatory dynamics origination in the populations with simple age structure and density dependent regulation of juveniles' survival. We have used classical methods for the systems stability study. To make numerical investigation of the model, we have elaborated the software systems, provided for construction of bifurcation diagrams, attraction basins, dynamic modes maps, and Lyapunov exponents. It is shown the density-dependent regulation of the offspring survival can lead to periodic and chaotic fluctuations in population size. The population parametric stability domain may considerably grow if the offspring survival rate would decrease with a number growth of both age groups. However, the regulation of juvenile survival rate by mostly the adults' number is inefficient and leads to the stability domain narrowing. Moreover, there are multistability areas in which the type of realized dynamic regime depending on the initial condition. In particular, the stable non-zero fixed point coexists with periodic points (3-cycle). These aspects of dynamic behaviour can explain the oscillation period change, appearance and disappearance of population size fluctuations.

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Solution to the Cauchy problem for parabolic PDE using space shift based formulas

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Quasi-stochastic Bykov attractors

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We consider a one-parameter unfolding of a symmetric three-dimensional vector field with a contracting Bykov network, while preserving the one-dimensional connection. When the parameter is zero, the dynamics is trivial and well known. After increasing the parameter, the stable and unstable manifolds of the saddle-foci intersect transversely, creating homoclinic and heteroclinic tangles. These are the source of a countable family of Smale horseshoes which, although not observable in numerical simulations, generate chaos inside a neighborhood of the network and prompt the coexistence of infinitely many sinks and saddle-type invariant sets.

In this talk, we clarify the role of both the stable/unstable manifolds of the singularities and the horseshoes in the overall structure and dynamics of the global attractor that exists for each parameter. We will see that for a dense set of parameters close enough to zero the dynamics has both infinitely many horseshoes and attracting periodic solutions. Moreover, the stable and unstable manifolds of both the equilibria and the horseshoes determine the non-wandering set of the flow. We present some numerical simulations to illustrate our results.

This is a joint work with M. Carvalho, M. Bessa and M. L. Castro.

Sectional-Anosov flows of venice mask type in compact 3-manifolds

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A basic result of the theory of hyperbolic dynamical systems is that every Anosov-flow with dense periodic orbits on a closed manifold is transitive. This fact is not true for sectional-Anosov flows (i.e., sets whose maximal invariant is a sectional-hyperbolic set) [6] due to existence of flows known as Venice masks [3,4]. I will expose some examples, consequences and I will mention some properties related to Venice masks supported on compact 3-dimensional manifolds [3,5].

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On the dynamics of quasiperiodically driven maps with weak dissipation

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We study the effect of external signal of incommensurate frequency on the dynamics of systems with weak dissipation. This situation is well studied for dissipative systems [1]. In particular, it is known that strange nonchaotic attractors (SNA) are typical in such systems. Also recently it was shown [2] that SNA exists in the system with weak dissipation but no results on the size of corresponding region on the parameter plane and its dependence on dissipation parameter is known.

We consider the Ikeda map [3] driven by signal with incommensurate frequency ("quasiperiodical signal"):

$$z_{n+1} = A(1 + \varepsilon \sin 2\pi\theta_n) + Bz_n e^{i|z_n|^2}, \theta_{n+1} = \theta_n + w.$$

where the frequency ratio w was taken equal to golden ratio $w = (\sqrt{5} - 1)/2$.

It is known that multistability is typical for systems with weak dissipation. We study the effect of driving on the mulstistability. Also we investigate the structure of the "nonlinearity parameter A - the driving amplitude ε" plane for different dissipation values up to very small ones by calculating the Lyapunov exponents.

We identify the SNAs as the regimes with negative largest Lyapunov exponent and strange structure. The structure of the attractors was studied by the rational approximants [1] method. It was revealed that the band of parameter A values in which the SNA exists decreases significanlty with the decrease of dissipation.

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Analysis the methods of implementation of the precision chaos generators

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In operation approaches on implementation of precision generators of chaotic oscillations on the basis of solid-state electronics are researched. The main attention is paid to the implementation methods of precision generators based on synthesis of ring self-oscillatory structures with adjustable parameters of back coupling.

The first and main part of the report is devoted to the widespread decomposition of the initial self-oscillatory system [1] described by equation $\dot{x} = f(x)$ with a vector $x \subset \mathbf{R}^n$ leading to couple of subsystems $x \to (p, q)$ such that $\dot{p} = \gamma(p, q)$ and $\dot{q} = \vartheta(p, q)$. Here $\gamma = f_k(x)$, $\vartheta = f_m(x)$ and $\{f_k(x) \mid x \in \{1, 2, \ldots, l\}\}$, $\{f_m(x) \mid x \in \{l+1, l+2, \ldots, r\}\}$. Within this approach the main criteria of a precision of ring chaotic oscillators are considered [2]. The role of concept of the synchronous chaotic response which is the major tool in study of the phenomenon of synchronization of chaotic systems is shown.

In the second part of the report oscillators to which it is impossible to apply the decomposition method are considered. As the most widespread type of similar systems three-point diagrams of generators are considered. Several possible options of synthesis of three-point precision generators of chaos based on use symmetric and specularly adding digraphs of the initial three-point diagrams are offered. The analysis of merits and demerits of the existing and offered methods is carried out.

The possibility of the analysis quality of a chaotic synchronous response by means of elements of symbolical dynamics is considered. On the example of RL-sequence [3] the method of an assessment quality of a chaotic synchronous response is shown.

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Duck factory on the two-torus: multiple canard cycles without geometric constraints

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Consider a generic slow-fast system on the two-dimensional torus

$$\begin{cases} \dot{x} = f(x, y, \varepsilon) \\ \dot{y} = \varepsilon g(x, y, \varepsilon) \end{cases} \qquad (x, y) \in \mathbb{T}^2 \cong \mathbb{R}^2/(2\pi\mathbb{Z}^2), \quad \varepsilon \in (\mathbb{R}_+, 0) \tag{9}$$

Assume that f and g are smooth enough and $g > 0$. The dynamics of this system is guided by the *slow curve*:

$$M = \{(x, y) \mid f(x, y, 0) = 0\}.$$

It consists of equilibrium points of the fast motion (i.e. the motion determined by system (9) for $\varepsilon = 0$). Particularly, one can consider two parts of the slow curve: one is stable (consists of attracting hyperbolic equilibrium points) and the other is unstable (consists of repelling hyperbolic equlibrium points). On the plane $\mathbb{R}^2$, there is rather simple description of the generic trajectory of (9): it consists of interchanging phases of the slow motion along the stable parts of the slow curve and the fast jumps along the straight lines $y = const$ near the folds of the slow curve [5]. On the two-torus, more complicated behaviour can be locally generic.

Definition 1. *A solution (or trajectory) is called* canard *if it contains an arc of length bounded away from zero uniformly in ε that keeps close to the unstable part of the slow curve and simultaniously contains an arc (also of length bounded away from zero uniformly in ε) that keeps close to the stable part of the slow curve.*

Canards are not generic on the plane: one have to introduce an additional parameter to get an attracting canard cycle. However, they are generic on the two-torus, as was conjectured in [1] and proved in [2]. The next natural question is to provide an estimate for the number of canard cycles that can born in a generic slow-fast system on the two-torus. The answer to this question for the case of integer rotation number and a rather wide class of systems was given in [3].

Theorem 1. *For generic slow-fast system on the two-torus with contractible nondegenerate connected slow curve the number of limit cycles that make one pass along the axis of the slow motion is bounded by the number of fold points of the slow curve. This estimate is sharp in some open set in the space of slow-fast systems on the two-torus.*

In the present talk we consider the case of non-integer rotation number that is not covered by Theorem 1. We also conjecture that our arguments can be applied to systems with unconnected slow curves. The latter case is of special interest because slow-fast systems with unconnected slow curve appear naturally in physical applications, e.g. in the modelling of circuits with Josephson junction [4].
Our main result states that in contrast with Theorem 1 for non-integer rotation number there are *no geometric constraints* on the number of (canard) limit cycles. Particularly, for any desired odd number of limit cycles l we construct open set in the space of the slow-fast systems on the two-torus with *convex* slow curve (i.e. having only two fold points) with exactly l canard cycles that make two passes along the axis of slow motion.

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Oscillatory motions in the restricted planar elliptic three body problem

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The Eccentric Kozai-Lidov Effect as a Resonance Phenomenon

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Considering the evolution of a weakly perturbed Keplerian motion under the scope of the restricted three-body problem M.L.Lidov (1961) and Y.Kozai (1962) discovered independently coupled oscillations of eccentricity and inclination (KL-cycles). Their classical investigations were based on

the integrable model of the secular evolution obtained after double averaging of the disturbing function approximated by the first non-trivial term (more precisely, by the quadruple term) in the series expansion with respect to the ratio of semimajor axis of the disturbed body and the disturbing body.
If the next (octupole) term is kept in the expression of the disturbing function, then the longterm modulation of the KL-cycles can be established (Ford et al., 2000; Katz et al., 2011; Lithwick, Naoz, 2011). In particular, the flips become possible from prograde to retrograde orbit and back again. Since flips are observed only in the case of the disturbing body motion in the orbit with non-zero eccentricity, the term "Eccentric Kozai-Lidov Effect" (or EKL-effect) was proposed in (Lithwick, Naoz, 2011) to specify such a dynamical behavior.
We demonstrate that the EKL-effect can be interpreted as a resonance phenomenon. With this aim we write down the motion equations in terms of the "action-angle" variables provided by the integrable Kozai-Lidov model. It turns out that for some initial values the resonance is degenerate and the usual "pendulum" approximation is insufficient to describe the evolution of the resonance phase. The analysis of the related bifurcations allows us to estimate the typical time between the successive flips for different parts of the phase space.

Volume hyperbolicity and wildness

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Complex dynamic regimes in the stochastic Hindmarsh-Rose model

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We study the stochastically forced Hindmarsh-Rose model [1] of neuron activity:

$$\begin{aligned} \dot{x} &= y - x^3 + 3x^2 + I - z + \varepsilon \dot{w}, \\ \dot{y} &= 1 - 5x^2 - y \\ \dot{z} &= r(s(x - x_0) - z), \end{aligned} \tag{10}$$

where x is a membrane potential, variables y, z describe ionic currents, I is an external current; $0 < r \ll 1$ is a time scale parameter; s, x_0 are other parameters; w is a standard Wiener process with $\mathrm{E}(w(t) - w(s)) = 0,\ \mathrm{E}(w(t) - w(s))^2 = |t - s|$ and ε is a noise intensity.
We fix $r = 0.002$, $s = 4$, $x_0 = -1.6$ and examine the dynamics of the system under variation of the parameter I.

The original deterministic system demonstrates a wide range of neural dynamic regimes, such as various types of periodic oscillations resulting from the period-doubling and adding bifurcations, coexistence of several attractors, chaos.

The aim of our research is is to study the effect of random disturbances on the dynamics of the Hindmarsh-Rose model. Here, we focus on the parametric zone, where the deterministic system exhibits tonic spiking oscillations. We show that under noise the spiking regime transforms into the bursting one. This phenomenon is confirmed by changes of distributions of random trajectories in the phase space and interspike intervals. We show that noise-induced bursting combines with transitions from order to chaos.

For a quantitative analysis of the stochastic phenomena in the Hindmarsh-Rose model, we suggest a constructive approach based on the stochastic sensitivity function technique [2] and the method of confidence domains. It gives a geometric description for distribution of random states around the deterministic attractors. We develop an algorithm of estimation of the threshold values for the noise intensity corresponding to the stochastic bifurcations. We show that the obtained values are in a good agreement with the numerical results.

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Variational construction of positive entropy invariant measures of Lagrangian systems and Arnold diffusion

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We develop a new variational method for constructing positive entropy invariant measures of Lagrangian systems without assuming transversal intersections of stable and unstable manifolds, and without restrictions to the size of perturbations. When applied to a family of 2 1/2 degrees of freedom a-priori unstable Hamiltonians, we obtain the following results, assuming only no topological obstruction to diffusion: (i) there exists a vast family of "horsheshoes", such as "shadowing" ergodic positive entropy invariant measures having precisely any closed set of invariant tori in its support; (ii) the drift acceleration is proportional to the average of the locally largest Lyapunov exponents (always non-zero in the region of instability) along a diffusion path; (iii) a sufficient condition for the fast diffusion with the acceleration drift $O(\mu/|\log \mu|)$ (μ is the size of the perturbation) is that the inverse of the splitting angles of separatrices is integrable, thus discrete non-transversal intersections of whiskers are not an obstacle for that; (iv) one can obtain lower bounds on local topological entropy and acceleration drift for Melnikov functions with degenerate zeroes, as a function of the leading term in its Taylor expansion at the zero.

The method of construction is new, and relies on study of formally gradient dynamics of the action (coupled parabolic semilinear partial differential equations on unbounded domains). We apply recently developed techniques of precise control of the local evolution of energy (in this case the Lagrangian action), energy dissipation and flux.

One-parameter families of vector fields on the two-dimensional sphere

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Suppose that a planar vector field X met in a generic one-parameter family has a separatrix loop, and the saddle value (trace of linearization) is positive. Suppose that this vector field has some saddles inside the separatrix loop, and exactly n separatrices tend to separatrix loop. Then on one side of the critical parameter value the field has one unstable hyperbolic cycle. On the other side of the critical value there is a countable number of bifurcation points related to saddle connections between two saddles. We state that any two generic one-parameter families $V = \{v_\varepsilon\}$ and $W = \{w_\varepsilon\}$ s.t. $v_0 = w_0$ *are equivalent*.

Ouasiperiodic bifurcations in five coupled van der Pol oscillators

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Dynamics of coupled oscillators attract intensive attention of researchers. Recently, enough big step was made in this area [1-2]. One of the important aspect of the dynamic of coupled oscillators is studying of multi-frequency quasi-periodic oscillations in the systems of such kind. The interest to this problems connect with opportunity of realization of Landau-Hopf scenario [3]. For realization Landau-Hopf scenario it is necessary to have quasi-periodic Hopf bifurcations at varying of the parameters. Factors which influent to the type of bifurcation are topology of coupling, distribution of base frequency of ensemble elements e.t.c.

In the present work dynamic of the system of five van der Pol oscillators coupled in a ring is considered. Opportunity of occurring of quasi-periodic Hopf bifurcation and saddle-node quasi-periodic bifurcation for five-frequency torus was revealed for system with active and dissipative coupling.

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On the dependence of solutions of convolution equations on convex sets of the right side

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Let Q be a convex subset of $\mathbf{C}$ with non-empty interior. We assume that Q has a countable neighborhood basis of convex domains. This setup covers in particular convex domains Q in $\mathbf{C}$ and convex compact sets Q in $\mathbf{C}$ with non-empty interior. Denote by $A(Q)$ the space of germs of all analytic functions on Q with its natural inductive limit topology. Let K be a convex compact set in $\mathbf{C}$. For a functional μ which is continuous and linear on $A(K)$ a continuous linear convolution operator

$$T_\mu : A(Q+K) \to A(Q)$$

is defined by $T_\mu(f)(z) := \mu_t(f(t+z))$. We characterize when surjective operator $T_\mu : A(Q+K) \to A(Q)$ has a continuous linear right inverse $R : A(Q) \to A(Q+K)$. Similar problem was posed in the early fifties by L. Schwartz for linear differential operators $P(D)$ in $C^\infty(\Omega)$ where Ω is an open subset of $\mathbf{R}^\mathbf{N}$. This problem was solved by R. Meise, B.A. Taylor and D. Vogt in the late eighties of the last century.

It is shown that R exists if and only if the distribution of zeros of the entire function $\hat{\mu}(z) := \mu_t(\exp(tz))$ satisfies certain conditions of the boundary behavior of the analytic univalent functions of the unit disc $\mathbf{D}$ on the interior of Q and of the complement of closed unit disc $\overline{\mathbf{D}}$ on the complement $\overline{Q}$. For example, if the boundary of Q of class C^2 each surjective operator $T_\mu : A(Q+K) \to A(Q)$ has a continuous linear right inverse.

Previously, this problem was solved when Q is a convex domain, a convex compact set and a convex locally closed set (see [1]).

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Analysis of spatial correlations at the coherence-incoherence transition in a ring of nonlocally coupled logistic maps

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We present numerical results for mutual spatial correlations of elements in a one-dimensional ring of nonlocally coupled chaotic maps. The global transition from complete chaotic synchronization to spatio-temporal chaos is studied when the coupling strength is varied. The logistic map in the regime of developed chaos is chosen as a partial element of the ring. The basic bifurcation transitions are analyzed for decreasing coupling strength and they range as follows: complete chaotic synchronization – partial chaotic synchronization – appearance of phase chimera states – birth of amplitude chimera states – complete desynchronization. The typical peculiarities in the behavior of mutual spatial correlations of the network elements are established for the indicated bifurcation transitions.

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Spectral gap for Lorenz 1D maps

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Chaotic systems are well known to be difficult to analyse if one looks at the behaviour of particular trajectories. However often if one considers the dynamics of the densities of measures that are absolutely continuous with respect to some natural measure, the analysis becomes feasible.

The dynamics of the densities is described by the action of the Perron-Frobenius operator. Its spectral properties are very related to the statistical properties of usual trajectories of the points.

In particular, spectral properties of Perron-Frobenius operators are important. I will describe how to prove so-called spectral gap property for a certain class of one dimensional piecewise expanding maps of small regularity, allowing power-law singularities of the derivative, on a large Banach space of observables of small regularity, allowing power-law singularities. Previous results in this direction required bigger regularity and/or smaller Banach space.
Such one-dimensional maps are much related to the famous Lorenz attractor (more precisely, to its geometric model).

From Regularity to Chaos in the GPE with Bichromatic Trap

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It is well known that BEC is described by Gross-Pitaevskii Equation (GPE) with an external potential. The nonlinear dynamic of GPE is very rich. Therefore many studies have been performed on nonlinear properties in GPE for different optical lattices. In this study, we consider the tilted bichromatic optical lattice potential that the sytem exhibits regular and chaotic dynamics under it. The regular and chaotic numerical solutions of GPE are investigated by constructing their Poincaré sections in phase space.

In search for the Hénon attractor

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An extensive search for stable periodic orbits (sinks) for the Hénon map in a small neighborhood of the classical parameter values is carried out. Several parameter values which generate a sink are found and verified by rigorous numerical computations. Each found parameter value is extended to a larger region of existence using a simplex continuation method. The structure of these regions of existence is investigated. This study shows that for the Hénon map, there exist sinks close to the classical case. This work is joint with Zbigniew Galias, AGH University, Krakow, Poland.

PROJECTIVE CONVERGENCE OF MEASURES

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I will speak about a distance for processes, based on Hibert's protective distance, which we recently introduced. I will explain how our distance compares to the *-weak distance and to Ornstein's d-bar distance. Among other things, we prove that our distance is strictly stronger than the *-weak distance, but it is not comparable with Ornstein's d-bar distance.
We also prove that all g-measure is the limit, with respect to our distance, of its Markovian approximations and how the ergodicity and mixing of the limiting g-measure is relate to the speed of convergence of the Markovian approximations.
I will finish by formulating some problems we are currently working on. This is a work in collaboration with Liliana Trejo and is based on a previous works in collaboration with Leticia Ramirez and Jean-Rene Chazottes.

Noise-induced transitions in a bistable oscillator with state-depended nonlinear dissipation

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A model of bistable oscillator with nonlinear dissipation depended on phase variables is proposed. This model possess a complex behavior with Gaussian noise excitation. We study its response to noise using a numerical simulation and an electronic circuit realization. The results show that depending on noise intensity the system undergoes multiple qualitative changes in the structure of its steady-state probability density function. In particular, the probability density function exhibits two pitchfork bifurcations with a noise intensity increasing. The behavior of noisy system can be described using an effective potential. These stochastic phenomena are explained, using the partition of the phase spaceby the nullclines of the system.

Splitting of separatrices at a periodically forced Hamiltonian-Hopf bifurcation

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On the dynamics of non-invertible branched coverings of surfaces

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Let $f : M \to M$ be a branched covering, i.e. an inner map of a surface M. Recall that an inner map is an open and isolated map. A map is open if the image of an open set is open. A map is isolated if the pre-image of a point consists of isolated points.

In [1] it was introduced a set of new invariants of topological conjugacy of non-invertible inner mappings that are modeled from the invariant sets of dynamical systems generated by homeomorphisms. Those new invariants are based on the analogy between the trajectories of a homeomorphism and the directions in the set of points having common image which is viewed as having 2 dimensions.

In the talk we explore the dynamical properties of wandering sets of different classes of branched coverings.

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The main asymptotics in the problem of Andronov-Hopf bifurcation

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Consider the differential equation

$$x' = A(\alpha)x + a(x, \alpha), \quad x \in R^N\,, \;\; N \geq 2\,, \tag{11}$$

where α - parameter, $A(\alpha)$ - matrix of order N, which elements smooth (continuously differentiable) depend on α, the nonlinearity $a(x, \alpha)$ can be represented in the form:

$$a(x, \alpha) = a_2(x, \alpha) + a_3(x, \alpha) + a_4(x, \alpha)\,;$$

here $a_2(x,\alpha)$ and $a_3(x,\alpha)$ contain quadratic and cubic in x terms and nonlinearity $a_4(x,\alpha)$ satisfies $\|a_4(x,\alpha)| = O(\|x\|^4)$ as $x \to 0$.

Suppose that the matrix $A(\alpha_0)$ has a simple eigenvalues $\pm\omega_0 i$. Let all other than $\pm\omega_0 i$ the eigenvalues matrix $A(\alpha_0)$ have negative real parts. Put $T_0 = 2\pi/\omega_0$.

The matrix $A(\alpha)$ is a continuous branch of eigenvalues of $\lambda(\alpha) = \tau(\alpha) + i\omega(\alpha)$ such that $\tau(\alpha_0) = 0$ and $\omega(\alpha_0) = \omega_0$. Let $\tau'(\alpha_0) \neq 0$. Then the number of α_0 is a point of Andronov-Hopf bifurcation equation (11). Thus, as a rule, there are functions

$$\alpha(\varepsilon) = \alpha_0 + \alpha_2\varepsilon^2 + O(\varepsilon^4),\ T(\varepsilon) = T_0 + T_2\varepsilon^2 + O(\varepsilon^4) \qquad (12)$$

such that for each small ε and $\alpha = \alpha(\varepsilon)$ system (11) has unstable periodic solution $x = x(t,\varepsilon)$ period $T(\varepsilon)$. Functions (12) is called the main asymptotes of bifurcational solutions of (11).

The report proposes new main asymptotes to study the problem of the Andronov-Hopf bifurcation, and also discusses some applications. As an application, we obtain new conditions for stability in the problem of Andronov-Hopf bifurcation equation (11).

Recent progress in the theory of inertial manifolds

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Multistability and multimodality in Ricker's model with genetically defined parameters*

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We investigated dynamic regims of uniform population and mechanisms of its stability loss. At that the size of considered population is described by Ricker's model with intrapopulation parameters defined on the genetic level under an influence of the natural selection:

$$\begin{cases} x_{n+1} = \bar{w}_n x_n \\ q_{n+1} = q_n\left(w_{AA}p_n + w_{Aa}(1-p_n)\right)/\bar{w}_n \end{cases} \qquad (13)$$

where $\bar{w}_n = w_{AA}(x_n)q_n^2 + 2w_{Aa}(x_n)q_n(1-q_n) + w_{aa}(x_n)(1-q_n)^2$ is the average fitness of the population in the n-th generation, x_n is the population number in n-th generation, and q_n is the frequency of allele A in the n-th generation. Genotype fitness (w_{ij}) decrease from the population size: $w_{ij} = exp(R_{ij}(1 - x_n/K_{ij}))$, where R_{ij} and K_{ij} are the Malthusian and resource parameters of the ij- genotype, respectively. R_{ij} and K_{ij} characterize the reproductive potential of a genotype and the capacity of the ecological niche, respectively. It has been shown that loss of stability by the nontrivial fixed points of system (1) occurs only by the period-doubling bifurcation provided that the model parameters are inside the biological meaningful region. Effects of multistability and multimodality arise in some parametric regions.

It has been shown, that along with a fixed point (dynamically stable or unstable) the circles of various length and chaotic attractors may exist in the model and it may be attractive for the system from some region of initial conditions. To measure the influence of initial conditions on the dynamics type and on the direction of the population evolution the basins of attraction for simultaneously existing asymptotic dynamic regimes has been constructed.

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On supporting manifolds for Morse-Smale systems without heteroclinic intersections

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We consider Morse-Smale systems without heteroclinic intersections on closed manifolds. In addition, for Morse-Smale flows, we assume the absence of periodic trajectories. In this case, we describe the topological structure of supporting manifolds. As a consequence, we get sufficient conditions of the existence of periodic trajectories.

We thank RFFI grant 15-01-03687, RNF grant 14-41-00044. This work was supported by the Basic Research Program at the National Research University Higher School of Economics in 2016.

Sliding modes for NLS-type lattice with saturable nonlinearity: discrete vs continuous description

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Discrete solitons (DS) in NLS-type lattice models have attracted significant attention from physicists and mathematicians. It is known that typically the energy of DS depends on its position on the lattice. Due to this, the motion of DS along the lattice is accompanied by radiation, therefore DS loses energy and eventually stops. The examples of lattices where this does not take place are rare. In these exceptional cases DS can travel in regime of *sliding mode*, without losses of energy.

Recently, it has been found numerically, [1], that NLS-type model with saturation

$$\frac{1}{h^2}\left(u_{n+1}-2u_n+u_{n-1}\right)+u-\frac{\theta u_n}{1+u_n^2}=0 \tag{1}$$

is a candidate for such a lattice. In the contribution, we address the features of the lattice (1). First, we give new numerical arguments that there exist a set $\{h_k\}_{k\in\mathbf{N}}$, such that the sliding modes are possible for each $h=h_k$. Second, we found numerically that the set $\{h_k\}_{k\in\mathbf{N}}$ has some remarkable properties. Specifically, h_k weakly depend on θ and the values $\tau_k\equiv 1/h_k$ are nearly equidistant. Third, we consider the continuous counterpart of (1),

$$\frac{h^2}{12}u_{xxxx}+u_{xx}+u-\frac{\theta u}{1+u^2}=0, \tag{2}$$

that is asymptotic to (1) as $h\to 0$. We found that (2) possesses a set of sliding modes for $\{h_k^*\}_{k\in\mathbf{N}}$. The values h_k obey an asymptotic rule as $k\to\infty$, such that $\tau_k^*\equiv 1/h_k^*$ are asymptotically equidistant, but the set $\{h_k\}$ differs essentially from the set $\{h_k^*\}$.

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Families of gap solitons in complex non-$\mathcal{PT}$-symmetric potentials

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Nonlinear Schrödinger Equation with additional potential $U(x)$

$$i\Psi_t = \Psi_{xx} - U(x)\Psi \pm |\Psi|^2 \Psi \qquad (1)$$

arises in various physical theories, such as nonlinear optics and theory of ultracold gases. Of particular interest are its solutions of the form $\Psi(x,t) = u(x) \cdot e^{i\omega t}$, where $u(x) \to 0$ as $x \to \pm\infty$ (stationary localized modes, LM). It is known that the LM can exist if $U(x)$ is the real periodic potential and $\omega \in \mathbf{R}$ lies in gaps of continuous spectrum of the operator $L = -d^2/dx^2 + U(x)$. Typically, these LM (also called in this case *gap solitons*, GS) form families that may be parametrized by $\omega \in \mathbf{R}$.

The situation changes when $U(x)$ becomes complex. Generically, one cannot expect the existence of GS for real ω. However, there are two remarkable cases when this may occur. One of them corresponds to $\mathcal{PT}$-symmetric potentials $U(x)$, i.e., such that $\overline{U(-x)} = U(x)$. Another possibility arises if $U(x)$ is a *Wadati potential*, i.e. there exists a real function $w(x)$ such that $U(x) = -\left[w^2(x) + iw_x(x)\right]$. Some examples of families of LM in these two cases have been given recently, however the GSs in the case of periodic Wadati potential remain poorly explored.

In the present contribution we consider the case of Wadati potential with

$$w(x) = \sin^2 x + A\sin 4x, \quad A \in \mathbf{R}$$

so that the corresponding $U(x)$ is complex but not $\mathcal{PT}$-symmetric. The function $u(x)$ in this case obeys a complex nonautonomous ODE of the second order:

$$u_{xx} + \left(\omega + w^2(x) + iw_x(x)\right)u \pm |u|^2 u = 0$$

We describe an efficient numerical method for the search of GSs in this case and report on continuous families of GSs parametrized by $\omega \in \mathbf{R}$ for Eq.(1).

Birkhoff Problem on the Depth of the Center for Skew Products of Maps of an Interval

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Let $I = I_1 \times I_2$ be a closed rectangle in the plane (I_1, I_2 are closed intervals), and $F : I \to I$ be a skew product of maps of an interval:

$$F(x, y) = (f(x), g_x(y)), \quad \text{where} \quad g_x(y) = g(x, y), \ (x; y) \in I.$$

The complete solution of Birkhoff problem is given for the depth of the center of skew products of maps of an interval from the space $\widetilde{T}^1_*(I)$, where $\widetilde{T}^1_*(I)$ is the space of C^1-smooth skew products of maps of an interval satisfying inclusion $F(\partial I) \subset \partial I$ for every $F \in \widetilde{T}^1_*(I)$ (here $\partial(\cdot)$ is a boundary of a set) and having Ω-stable (in C^1-norm) quotient map. Considerations are based on the use of Decomposition Theorem for the space $\widetilde{T}^1_*(I)$ (see [1]).

This work is supported by Russian Science Foundation, grant 16-11-10036.

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ABOUT POSSIBILITY OF GENERATION OF HIGH-FREQUENCY ELECTRO-MAGNETIC FIELD IN EXCITABLE TISSUES

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It is known. what is the properties of biotissue are defined by its cells, and electrical work of cell - their biomembranes therefore the behavior of excitable biomembrane (EBM) as the main source of electromagnetic radiation is considered.

Along with use of the canonical differential equations of Hodzhkin-Huxley of the fourth order (and their modifications), Ziman's equations of the third order for the description and the analysis of work of EBM the equations like Van-der-Pol-Duffing (VDPD) of the second order are used (Fitz-Hugh, 1961).

Systems like VDPD are applied to convenience of the analysis of the movement of variables (in particulars, currents and tension in EBM) also give good similarity in form. amplitude, porosity with really measured parameters of impulses in EBM.

However, for example, in fundamental work Fitz-Hugh time interval in behavior of variables at generation of self-oscillations are not specified, and frequency characteristics of the corresponding VDPD model were not considered.

In this regard, consideration of range of frequencies of fluctuations in system of VDPD in the self-oscillatory mode for assessment of adequacy of VDP model to the real movements of variables in EBM is obviously important.

For such analysis of systems of VDPD it is convenient to pass to equivalent circuit like the generator on the tunnel diode (GTD) as the movements of currents and tension t is described completely by the similar equations.

Frequency of self-oscillations of fluctuations depends on ratio of losses (R-resistance of losses of membrane transition), pumpings (E - membrane e.d.s.) and sizes negative resistance on the falling site of the volt-ampere characteristic (VAC) of the tunnel diode, or, otherwise, from corner between load straight line and tangent to the falling site VAC in point of intersection.

Dependence of the period of fluctuations (in parametrical space of stability), for example, from R where there is upper limit of the period of Tmax that there corresponds ~ 1 ms, or ~ 1 kHz was considered analytically.

Thus, during the work of GTD (with real biophysical values of parametres) in space of steady self-oscillations frequency cannot be lower ~ than 1 kHz and there is in range ~ (1 - 10000) kHz.

It means that though the model Fitz-Hugh is quantitatively not adequate at model operation of cardiograms, during the work of VBM the electromagnetic field high-pitched (HF) which can exert particular impact on functioning of excitable and unexcitable tissues alive electrically can be excited. Extent of such influence can be the considerable since amplitude of the corresponding fluctuations ~ 1 mV, that is an order of aмplitudes of the cardiograms of heart.

IDENTIFICATION OF CHAOTIC SIGNALS BY THE FORM THE LAW OF DISTRIBUTION OF DENSITY OF PROBABILITY OF SELECTIVE VALUES

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The summary. In work identification of a chaotic signal of generator Chua by the form the law of distribution of density of probability of selective values is considered.
Key words: identification, chaotic, the generator, a kind of distribution, sample.

Generator Chua is the elementary dynamic system in which there can be a dynamic chaos. Practically to realize this system and consequently also chaotic fluctuations it is possible, using the generator of harmonious fluctuations or the oscillatory contour consistently connected and closed in ring system with the filter of the bottom frequencies, loaded on nonlinear resistance [1].

At studying the dynamic systems which are finding out irregular behaviour of decisions of the differential equations, there is a question connected to an establishment of the fact of existence of chaotic fluctuations. The various methods connected to the decision and the analysis of the differential equations [1] are applied to the decision of a task in view.

In the given work the method of identification of a chaotic signal by the form the law of distribution of his selective values is offered. For this purpose the algorithm based on calculation of statistical characteristics of selective values of a signal from an output of generator Chua is used. As parameter of identification it is offered to use the dimensionless parameter Z calculated under the formula [2]:

$$Z = \frac{k_э}{¡} + 4s, \qquad (1)$$

where ¡ - a counterexcess, $k_э$ - entropic parameter,, s - asymmetry parameter/

During researches mathematical modelling the chaotic generator was carried out. The kind of distribution of a signal of the generator was identified as arc sin interference.

For check of adequacy of modelling the circuit of generator Chua on the tunnel diode [3] has been practically realized. With the help of eight digit analog-to-digital converter with frequency of digitization 27 kGz the signal of the generator was digitized, the received readout entered the name in a digital file which was used by the program of the identification realizing algorithm (the formula (1)).

The result of identification of a kind of distribution has coincided with result of modelling. The density of probability had arksinuny distribution. At submission on an input of the generator of a harmonious signal the kind of distribution changed.

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Feynman integrals and radiation of waves in two- and three-dimensional spaces

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In report [1] we have found that one-dimensional plane wave can be expressed as Feynman integral of the next kind:

$$\frac{\exp(i\cdot x\cdot ¶)}{\sqrt{-2\cdot ®\cdot i}} = \int_{Q(-®/2)=¶}^{Q(0)=x} \exp\left[-i\cdot\int_{-®/2}^{0}\left(P(t)\cdot\dot{Q}(t)-\frac{P^2(t)+Q^2(t)}{2}\right)\cdot dt\right]\cdot d« , \quad (1)$$

where $d« = \prod_t \frac{dP(t)\cdot dQ(t)}{2\cdot ®}$ is Feynman pseudomeasure.

Furthermore using Green’s function of asymmetrical singular oscillator from paper [2] we establish the following expression for Bessel functions:

$$\sqrt{x\cdot ¶}\cdot J_{\nu}(x\cdot ¶) = \int_{Q(0)=¶}^{Q(®/2)=x} \exp\left[i\cdot\int_{0}^{®/2}\left(P(t)\cdot\dot{Q}(t)-\frac{P^2(t)+Q^2(t)}{2}-\frac{4\cdot ¬^2-1}{8\cdot Q^2(t)}\right)\cdot dt\right]\cdot d« . \quad (2)$$

Let us consider expansions of two- and three-dimensional plane waves [3]:

$$\exp(i\cdot\vec{q}\cdot ^{\sigma}) = \sum_{m=-\infty}^{\infty} i^m\cdot J_m(q\cdot °)\cdot\exp[i\cdot m\cdot ¤] , \quad (3)$$

where ¤ is angle between vectors $\vec{q}\in R^2$ and $^{\sigma}\in R^2$, and

$$\frac{\exp(i\cdot\vec{k}\cdot\vec{r})}{4\cdot ®} = \sqrt{\frac{®}{2\cdot k\cdot r}}\cdot\sum_{l=0}^{\infty}\sum_{m=-l}^{l} J_{l+\frac{1}{2}}(k\cdot r)\cdot Y_{lm}^{*}\left(\frac{\vec{k}}{k}\right)\cdot Y_{lm}\left(\frac{\vec{r}}{r}\right) , \quad (4)$$

where $Y_{lm}(\ldots)$ are spherical harmonics [3] and vectors $\vec{k}\in R^3$ and $\vec{r}\in R^3$.

Substituting formulae (1) and (2) into formulae (3) and (4) we obtain untrivial connections between Feynman integrals (1) and (2). One can understand these connections as new example of application of nonstandard analysis [4].

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О трехмерных диффеоморфизмах с одномерными базисными множествами, просторно расположенными на 2-торах.

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В докладе рассматривается класс G структурно устойчивых диффеоморфизмов, заданных на трехмерном замкнутом многообразии M^3 и удовлетворяющих следующим условиям:

1. неблуждающее множество диффеоморфизма $f \in G$ расположено на объединении $T_1 \cup T_2 \cup \cdots \cup T_k$ конечного числа $k \geq 2$ попарно непересекающихся f-инвариантых ручно вложенных в M^3 двумерных торов;

2. каждый тор T_i содержит в точности одно одномерное просторно расположенное базисное множество Ω_i диффеоморфизма f;

3. каждая компонента связности дополнения $T_i \setminus \Omega_i$ содержит в точности одну периодическую точку диффеоморфизма f.

Хорошо известно [1], что диффеоморфизм $f : M^3 \to M^3$ является структурно устойчивым, если он удовлетворяет аксиоме A С. Смейла (которая заключается в гиперболичности неблуждающего множества $NW(f)$ и плотности в нем множества периодических точек) и удовлетворяет строгому условию трансверсальности.

В силу теоремы о спектральном разложении С. Смейла неблуждающее множество диффеоморфизма, удовлетворяющего аксиоме A, состоит из объединения замкнутых непересекающихся базисных множеств, каждое из которых содержит транзитивную орбиту.

Напомним, что пара чисел $(dimW^u_x, dimW^s_x)$ не зависит от выбора точки x, принадлежащей базисному множеству Ω_i, и называется его типом.

Напомним также, что базисное множество диффеоморфизма, удовлетворяющего аксиоме A, называется растягивающимся аттрактором (сжимающимся репеллером), если его размерность совпадает с размерностью неустойчивых (устойчивых) многообразий его точек. Одномерный аттрактор (репеллер) всегда является растягивающимся (сжимающимся). Для диффеоморфизма из рассматриваемого класса

G аттрактор (репеллер) имеет тип $(1,2)$ ($(2,1)$. Если же одномерное базисное множество не является ни растягивающимся аттрактором ни сжимающимся репеллером, то оно является седловым базисным. При этом, если его тип есть $(2,1$ то оно принадлежит объединению неустойчивых многообразий его точек, а если его тип есть $(1,2)$, оно принадлежит объединению устойчивых многообразий его точек.

Основным результатом настоящего доклада является следующая теорема.

Теорема Неблуждающее множество $NW(f)$ диффеоморфизма $f \in G$ либо содержит $\frac{k}{2}$ одномерных аттракторов и $\frac{k}{2}$ седловых базисных множеств типа $(2,1)$, либо содержит $\frac{k}{2}$ одномерных репеллеров и $\frac{k}{2}$ седловых базисных множеств типа $(1,2)$.

В работе [2] построен класс Φ модельных диффеоморфизмов на многообразии $M_{\widehat{J}} = \mathbb{T}^2 \times \mathbb{R}/_\sim$, где $(z,r) \sim (\widehat{J}(z), r-1)$ для некоторого алгебраического автоморфизма $\widehat{J}$ тора $\mathbb{T}^2$, заданного матрицей $J \in GL(2,\mathbb{Z})$, которая либо является гиперболической, либо $J = \pm Id$. Каждый диффеоморфизм $\phi \in \Phi$ локально является прямым произведением алгебраического автоморфизма тора $\mathbb{T}^2$, заданного гиперболической матрицей $C \in GL(2,\mathbb{Z})$, такой что $CJ = JC$, и структурно устойчивого диффеоморфизма окружности $\mathbb{S}^1$. Доказано, что каждый диффеоморфизм f из класса G полусопряжен некоторому модельному диффеоморфизму $\phi \in \Phi$. В настоящей работе найден полный топологический инвариант и доказаны необходимое и достаточное условия топологической сопряженности двух диффеоморфизмов из класса G.

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[1] Robinson C. Structural stability of C^1-diffeomorphisms. // J. Differential Equations, 22, 28-73 (1976).

[2] Левченко Ю.А., Шиловская А.А. Диффеоморфизмы 3-многообразий с одномерными базисными множествами, просторно расположенными на 2-торах // Журнал СВМО. – №3, 2013. – С. 108–111.