Mutual synchronization of two spin transfer nano-oscillators

Aleshin K.N., Matrosov V.V., Mishagin K.G.

Lobachevsky State University of Nizhni Novgorod

kirill_al@bk.ru

Spin transfer nano-oscillators (STNO) are promising nano-scale devices that allow producing of ultra high frequency signals in range form 1 to 100 GHz and higher. Typical nano-oscillator is constructed from several ferromagnetic layers separated by non-ferromagnetic layers. Magnetization of one of the layers (free layer) is driven by three factors: 1) Larmor precession about direction of effective magnetic field; 2) damping due to energy dissipation (i.e. exchange coupling); 3) spin transfer via spin-polarized electric current that flows through the multilayer.

Direct electrical current applied to the STNO could cause spin wave excitation in free layer [1] so two STNO placed on common multilayer structure become coupled via spin waves.

Theoretical model for two coupled STNO via spin waves is expressed in terms of complex amplitudes of spin wave modes [1]:

$$\begin{aligned}\frac{dc_1}{dt} &= -i\omega_1 c_1 - (\eta_1 - \beta_1 J_1)c_1 - iT_1\alpha_1 n_1 c_1 - \frac{3\beta_1 J_1}{2SN} \\ &\quad \times (u_1^2 + v_1^2) n_1 c_1 - i\frac{2}{3}T_1\lambda^2(\delta_1 n_2 c_1 + \delta c_1^* c_2 c_2) \\ &\quad - \frac{\beta_1 J_1}{SN}\lambda^2[2(u_2^2 + v_2^2)n_2 c_1 + u_2^2 c_1^* c_2 c_2], \qquad (1)\end{aligned}$$

where c_1 – complex amplitude of the 1-st mode of spin wave, $n_1 = |c_1|^2$, ω_1 – it's natural frequency, η_1 – damping parameter, β_1 – spin-transfer term, J_1 – electric current density, λ – coupling parameter. Other parameters are discussed in [2]. Equation for second mode derived from (1) mutually switching indexes 1 and 2.

Change of coordinate system of the form $c_{1,2} = \rho_{1,2} \exp \varphi_{1,2}$ leads to equations:

$$\begin{aligned}\frac{d\rho_1}{d\tau} &= -(a_2 + a_3\rho_1^2)\rho_1 - \lambda^2\rho_2^2\rho_1(a_{21} - a_{12}\sin\theta + a_{22}\cos\theta), \\ \frac{d\rho_2}{d\tau} &= -(b_2 + b_3\rho_2^2)\rho_2 - \lambda^2\rho_1^2\rho_2(b_{21} + b_{12}\sin\theta + b_{22}\cos\theta), \\ \frac{d\theta}{d\tau} &= -2[(\gamma + b_1\rho_2^2 - a_1\rho_1^2) + \lambda^2\rho_2^2(a_{12}\cos\theta + a_{11} + a_{22}\sin\theta) -\end{aligned}$$

$$-\lambda^2\rho_1^2(b_{12}\cos\theta + b_{11} - b_{22}\sin\theta)], \qquad (2)$$

where $\rho_{1,2}$ – real amplitudes of the mode, $\theta = \varphi_2 - \varphi_1$ – phase difference between 1-st and 2-nd modes, $\gamma = (\omega_2 - \omega_1)/\omega_1$. Parameters a_i and b_i calculated corresponding to parameters in (1), their numerical values are given by real experiment data [2]. This form reduces topological dimension of the original system (1) and allows deeper understanding of the dynamical regimes.

In current work we present results of the full bifurcational analysis of system (2) and check correspondence between dynamical regimes of (1) and (2).

[1] Slonczewski, John C. "Current-driven excitation of magnetic multilayers." Journal of Magnetism and Magnetic Materials 159.1-2 (1996): L1-L7.

[2]Rezende, S. M., et al. "Mode locking of spin waves excited by direct currents in microwave nano-oscillators." Physical review letters 98.8 (2007): 087202.

Characteristics and Properties of Chimera States in Ensembles of Coupled Oscillators

V. Anishchenko

Saratov Univ., Russia

wadim@info.sgu.ru

Spiral chaos in a predator-prey model with disease in the predator

Bakhanova Yu.V.

Lobachevsky State University of Nizhny Novgorod

jul95-8@mail.ru

In this work we present some results of the investigation of chaotic behavior in a predator-prey model with disease in the predator [1]. We show that chaotic attractor in this system appears due to Shilnikov scenario [2]: stable equilibrium -> (Andronov-Hopf bifurcation) -> stable limit cycle + sadle-focus equilibrium -> unstable invariant manifold of saddle-focus winds on the stable limit cycle -> the limit cycle loses its stability (due to cascade of period doubling bifurcations) -> the unstable invariant manifold touches one-dimensional stable invariant manifold and forms homoclinic trajectory to the saddle-focus with negative saddle value. In addition we calculate diagrams of maximal Lyapunov exponents and found 2 periodicity hubs - codimension two points to which bifurcation curves of stable limit cycles and the regions with chaotic behavior accumulate by a spiral way [3]. We show that these 2 hubs "connected" by the bifurcation curves of the homoclinic trajectories to the saddle-focus.

[1] Krishna pada Das. A study of chaotic dynamics and its possible control in a predator prey model with disease in the predator. Journal of Dynamical and Control Systems, 21(4) (2002), 605-624.

[2] L. P. Shilnikov. The theory of bifurcations and turbulence. Selecta Math. Sovietica, 10 (1991), 43-53.

[3] Vitolo R., Glendinning P., Gallas J. A. C. Global structure of periodicity hubs in Lyapunov phase diagrams of dissipative flows. Physical Review E, 84(1) (2011), 016216.

Arabesque Chaos: Circuit Analysis and Chaos Detection

V. Basios

University of Brussels, Belgium

vbasios@ulb.ac.be

Complex dynamics of pedestrian-bridge interactions

Igor Belykh[1], Russell Jeter[1], and Vladimir Belykh[2]

[1] *Georgia State University, Atlanta, USA*

[2] *Volga State University of Water Transport, Nizhny Novgorod, Russia*

ibelykh@gsu.edu

Several modern footbridges around the world have experienced large lateral vibrations during crowd loading events. The onset of large-amplitude bridge wobbling has generally been attributed to crowd synchrony; although, its role in the initiation of wobbling has been challenged [1]. To study the contribution of a single pedestrian into overall, possibly unsynchronized, crowd dynamics, we use a bio-mechanically inspired inverted pendulum model of human balance and analyze its bi-directional interaction with a lively bridge [2]. Through theory and numerics, we demonstrate that pedestrian-bridge interactions can induce bistable lateral gaits such that switching between the gaits can initiate large-amplitude wobbling. We also analyze the role of stride frequency and the pedestrian's mass in hysteretic transitions between the two types of wobbling. Our results support a claim that the overall foot force of pedestrians walking out of phase can cause significant bridge vibrations.

1. Igor Belykh, Russell Jeter, and Vladimir Belykh, "Foot force models of crowd dynamics on a wobbly bridge," (submitted to Science Advances), e-print: arXiv: 1610.05366.

2. Igor Belykh, Russell Jeter, and Vladimir Belykh, "Bistable gaits and wobbling induced by pedestrian-bridge interactions," Chaos, V. 26, 116314 (2016).

Homoclinic bifurcations and attractors in certain maps and flows

V. Belykh

Lobachevsky State University of Nizhny Novgorod

belykh@unn.ac.ru

Homogenization of random trajectory attractors for reaction–diffusion systems

Chechkin G.A.

M.V.Lomonosov Moscow State University, Russia

chechkin@mech.math.msu.su

Chepyzhov V.V.

Institute for Information Transmission Problems, RAS, Russia;

National Research University Higher School of Economics, Russia

chep@iitp.ru

In this work we study asymptotic behavior of trajectory attractors of autonomous reaction–diffusion systems with randomly oscillating terms. To study such phenomenon we apply the homogenization method (cf., for example, [1],[2],[3], for random case cf., for instance, [4] as well as a delicate analysis of attractors for dissipative partial differential equations (see, for example, [5],[6],[7] and the references therein).

Assume that $(\Omega, \mathcal{B}, \mu)$ is a probability space, i.e., the set Ω is endowed with a σ-algebra $\mathcal{B}$ of its subsets and a σ-additive nonnegative measure μ on $\mathcal{B}$ such that $\mu(\Omega) = 1$.

We consider reaction-diffusion systems with randomly oscillating terms of the form

$$\partial_t u = a\Delta u - b\left(x, \frac{x}{\varepsilon}, \omega\right) f(u) + g\left(x, \frac{x}{\varepsilon}, \omega\right),\ u|_{\partial D} = 0, \tag{3}$$

where $x \in D \Subset \mathbb{R}^n,\ u = (u^1, \ldots, u^N),\ f = (f^1, \ldots, f^N)$, and $g = (g^1, \ldots, g^N)$. Here a is an $N \times N$ matrix with positive symmetric part and $b(x, z, \omega) \in C(D \times \mathbb{R}^N \times \Omega)$ is a real positive function. For the simplicity we assume that the vector function $f(v) \in C(\mathbb{R}^N; \mathbb{R}^N)$ satisfies the following inequalities:

$$f(v) \cdot v \geq \gamma |v|^p - C,\ |f(v)| \leq C_1 \left(|v|^{p-1} + 1\right),\ p \geq 2. \tag{4}$$

Notice that we *do not assume* that the function $f(v)$ satisfies Lipschitz-type condition with respect to v. This means that the uniqueness theorem for the Cauchy problem of system (3) may not hold.

Assume that $T_\xi, \xi \in \mathbb{R}^n$, is an *ergodic dynamical system* on Ω. Let the function $b(x, \frac{x}{\varepsilon}, \omega)$ and vector function $g(x, \frac{x}{\varepsilon}, \omega)$ be *statistically homogeneous*, i.e. $b(x, \xi, \omega) = \mathbf{B}(x, T_\xi \omega)$ and $g(x, \xi, \omega) = \mathbf{G}(x, T_\xi \omega)$, where $\mathbf{B} : D \times \Omega \to \mathbb{R}$ and $\mathbf{G} : D \times \Omega \to \mathbb{R}^N$ is measurable. We also assume that $b(x, z, \omega) \in C_b(\overline{D} \times \mathbb{R} \times \Omega)$ and

$$\beta_1 \geq b(x, z, \omega) \geq \beta_0 > 0,\ \forall x \in D,\ z \in \mathbb{R}^n,\ \omega \in \Omega, \tag{5}$$

and the function $b\left(x,\frac{x}{\varepsilon},\omega\right)$ has the average $b^{hom}(x)=\mathbb{E}(\mathbf{B})(x)$ as $\varepsilon\to 0+$ in $L_{\infty,*w}(D)$, that is, almost surely

$$\int_D b\left(x,\frac{x}{\varepsilon},\omega\right)\varphi(x)dx \to \int_D b^{hom}(x)\,\varphi(x)dx\ (\varepsilon\to 0+),\ \forall\ \varphi\in L_1(D). \tag{6}$$

For the vector function $g\left(x,\frac{x}{\varepsilon},\omega\right)$ we also assume that it has the average $g^{hom}(x)=\mathbb{E}(\mathbf{G})(x)$ in the space $V'=\left(H^{-1}(D)\right)^N$, that is, almost surely

$$\left\langle g\left(x,\frac{x}{\varepsilon},\omega\right),\varphi(x)\right\rangle \to \left\langle g^{hom}(x),\varphi(x)\right\rangle\ (\varepsilon\to 0+),\ \forall\ \varphi\in V=\left(H_0^1(D)\right)^N. \tag{7}$$

Denote the space $H=\left(L_2(D)\right)^N$. We consider weak solutions (trajectories) of the system (3), that is, the functions $u(x,t)\in L_\infty^{loc}(\mathbb{R}_+;H)\cap L_2^{loc}(\mathbb{R}_+;V)\cap L_p^{loc}\left(\mathbb{R}_+;(L_p(D))^N\right)$ which satisfy (3) in the sense of distributions. We denote by $\mathcal{K}_\varepsilon^+$ the set of all weak solutions of the system (3). Consider the *translation semigroup* $\{T(h)\}$ acting on the *trajectory space* $\mathcal{K}_\varepsilon^+$ by the formula $T(h)u(x,t)=u(x,t+h)$ for $h\geq 0$.

We study the (strong) *trajectory attractor* $\mathfrak{A}_\varepsilon$ of the system (3) that, by definition, coincides with the global $\left(\mathcal{F}_+^b,\Theta_+^{s,loc}\right)$-attractor of the translation semigroup $\{T(h)\}$ acting on $\mathcal{K}_\varepsilon^+$ (see [5],[6],[7]). Here, $\Theta_+^{s,loc}$ denotes the local *STRONG* topology, which is determined by the local strong convergence of sequences $\{v_m\}$ and $\{\partial_t v_m\}$ in the corresponding spaces. The trajectory space $\mathcal{K}_\varepsilon^+$ is supplied with topology $\Theta_+^{s,loc}$. The Banach space $\mathcal{F}_+^b$ is used to define bounded sets in $\mathcal{K}_\varepsilon^+$.

Along with the random system (3) we consider the averaged deterministic system

$$\partial_t\bar{u}=a\Delta\bar{u}-b^{hom}(x)\,f(\bar{u})+g^{hom}(x),\ \bar{u}|_{\partial D}=0. \tag{8}$$

System (8) also has the strong trajectory attractor $\overline{\mathfrak{A}}$ in the trajectory space $\overline{\mathcal{K}}^+$ corresponding to the system (8).

Theorem. *The following limit holds almost surely in the local strong topology* $\Theta_+^{s,loc}$

$$\mathfrak{A}_\varepsilon\to\overline{\mathfrak{A}}\ \text{ as } \varepsilon\to 0+\,. \tag{9}$$

Let $\operatorname{dist}_{\mathcal{M}}(X,Y):=\sup_{x\in X}\operatorname{dist}_{\mathcal{M}}(x,Y)$ denote the Hausdorff semidistance from a set X to a set Y in a metric space $\mathcal{M}$.

Corollary. *For any $M > 0$ we have almost surelly in Ω*

$$\begin{aligned}\operatorname{dist}_{L_2([0,M];H^1)}\left(\Pi_{0,M}\mathfrak{A}_\varepsilon, \Pi_{0,M}\overline{\mathfrak{A}}\right) &\to 0 \ (\varepsilon \to 0+),\\ \operatorname{dist}_{C([0,M];H)}\left(\Pi_{0,M}\mathfrak{A}_\varepsilon, \Pi_{0,M}\overline{\mathfrak{A}}\right) &\to 0 \ (\varepsilon \to 0+).\end{aligned}$$

Remark. Analogous theorem holds for random non-autonomous reaction-diffusion systems of the form (3) which contain the terms $b\left(x, \frac{t}{\varepsilon}, t, \omega\right)$ *and* $g\left(x, \frac{t}{\varepsilon}, t, \omega\right)$ *having the uniform averages in time as* $\varepsilon \to 0+$.

The work is partially supported by the Russian Foundation of Basic Researches (projects no. 17-01-00515 and 15-01-07920) and the Russian Science Foundation.

[1] Bensoussan A., Lions J.-L., Papanicolau G. Asymptotic Analysis for Periodic Structures, North–Holland, Amsterdam (1978).

[2]Bakhvalov N.S., Panasenko G.P. Averaging Processes in Periodic Media, Kluwer Academic Publ., Dordrecht (1989).

[3]Jikov V.V., Kozlov S.M., Oleinik O.A. Homogenization of Differential Operators and Integral Functionals, Springer–Verlag, Berlin etc. (1994).

[4]Amirat Y., Bodart O., Chechkin G.A., Piatnitski A.L. Boundary homogenization in domains with randomly oscillating boundary. Stochastic Processes and their Applications. 2011. V. 121, No 1. P. 1–23.

[5]Babin A.V., Vishik M.I. Attractors of evolution equations. North–Holland, Amsterdam (1992); Nauka, Moscow (1989).

[6]Chepyzhov V.V.,Vishik M.I. Attractors for equations of mathematical physics. Amer. Math. Soc. Providence, RI (2002).

[7]Temam R. Infinite-dimensional dynamical systems in mechanics and physics. Applied Mathematics Series. V. 68, Springer-Verlag, New York (1988).

Existence of global weak solutions to a hybrid Vlasov-MHD model for magnetized plasmas

B. Cheng

University of Surrey, UK

b.cheng@surrey.ac.uk

On quasiperiodic perturbations of the Duffing equation

T. Dragunov, A. Morozov

Lobachevsky State University of Nizhny Novgorod

timofey.dragunov@itmm.unn.ru, morozov@mm.unn.ru

Splitting of separatrices, scattering maps and energy growth for a billiard inside a time-dependent symmetric domain close to an ellipse

Fain V, Dettmann C P, Turaev V

University of Bristol

vf13950@bristol.ac.uk

We study billiard dynamics inside an ellipse with a time-dependent periodic perturbation of the axes and small $O(\delta)$ quartic polynomial deformation of the boundary. In this situation the energy of the billiard is no longer conserved. For high initial energy E^*, the billiard map may be considered as a small and slow perturbation of a static elliptic billiard in terms of the perturbation parameters $\frac{1}{\sqrt{E^*}}$ and δ. We prove a type of Fermi acceleration, namely that for any $\delta > 0$, given a sufficiently large initial energy, there exists a billiard trajectory that reaches an arbitrarily larger energy value. The proof depends on reduction of the billiard map to two Hamiltonian flows defined on the normally hyperbolic invariant manifold Λ parametrised by energy and time in the phase space of the billiard. The two flows approximate inner and scattering maps, which are common tools that arise in the studies of Arnol'd diffusion. Melnikov type calculations imply that in the first order in the perturbation parameters, the foliations $W^{ss,uu}$ of the stable and unstable invariant manifolds $W^{s,u}(\Lambda)$ of Λ are *nontransverse* to $W^{s,u}$ at $\delta = 0$. The consequence of this is that the *scattering map* is only defined on a subset of Λ that increases with δ and becomes empty for $\delta = 0$.

Two-dimensional invariant tori in a chain of coupled FitzHugh-Nagumo type oscillators

Glyzin S. D.[*,], Kolesov A. Yu.[*], Rozov N. Kh..[***]**

[*] *P.G. Demidov Yaroslavl State University,* [**] *Scientific Center in Chernogolovka RAS*

[***] *M.V. Lomonosov Moscow State University*

glyzin.s@gmail.com, kolesov@uniyar.ac.ru, fpo.mgu@mail.ru

A mathematical model of a one-dimensional array of FitzHughNagumo neurons with resistive-inductive coupling between neighboring elements is proposed. The model relies on a chain of diffusively coupled three-dimensional systems of ordinary differential equations

$$\begin{aligned} \dot{u}_j &= v_j + \lambda(u_j - u_j^3/3), \\ \dot{v}_j &= -u_j + d\,(u_{j+1} - 2u_j + u_{j-1}) - \varepsilon w_j - \varepsilon\beta\,(v_j - w_j), \\ \dot{w}_j &= -u_j - \varepsilon w_j, \end{aligned} \tag{1}$$

where

$$j = 1, 2, \ldots, N, \quad u_0 = u_1, \quad u_{N+1} = u_N, \tag{2}$$

System (1) – (2) is the mathematical model of the FitzHughNagumo neural network.

Under conditions $0 < \varepsilon \ll 1$, $\lambda = \varepsilon\alpha$, where $\alpha, \beta, d = \text{const} > 0$ we prove that any fixed number of coexisting stable two-dimensional invariant tori can be obtained in this system by suitably increasing the parameters α and N. To conclude, we note that there is an intermediate variant when both the number of coexisting stable cycles and the number of stable tori (more precisely, three-dimensional tori) grow indefinitely as $\alpha, N \to \infty$ and $\varepsilon \to 0$. With the use of the technique developed in [1], it can be shown that the same situation takes place in system (1) with periodic boundary conditions $u_{j+2N} = u_j$, $j \in \mathrm{Z}$.

Acknowledgments. This work was supported by the Russian Science Foundation (project nos. 14-21-00158).

1. A.Yu. Kolesov, N.Kh. Rozov, Dynamic effects associated with spatial discretization of nonlinear wave equations, Comput. Math. Math. Phys.,49:10 (2009), 17331747.

Strange attractors and scenarios of their appearance in 3-dimensional diffeomorphisms

Gonchenko A.S.

Nizhny Novgorod State University, Russia

agonchenko@mail.ru

For one-parameter families of three-dimensional orientable and nonorientable maps, we study scenarios of appearance of strange homoclinic attractors (containing only one fixed point). We describe several such scenarios. Some examples of realization of these scenarios in the case of three-dimensional orientable and nonorientable generalized Hénon maps are given. We consider three-dimensional generalized Hénon maps of form $\bar{x} = y,\ \bar{y} = z,\ \bar{z} = Bx + Az + Cy + g(y, z)$, where A, B and C are parameters (B is the Jacobian) and $g(0, 0) = g'(0, 0) = 0$.

1:4 resonance in the conservative cubic Hénon maps

M. Gonchenko

Barcelona University, Spain

mgonchenko@gmail.com

We study local and global aspects of bifurcation of 1:4 resonance in the conservative cubic Hénon maps which appear as truncations of rescaled first return maps near cubic homoclinic tangencies for two-dimensional symplectic maps with a saddle fixed point. We prove that 2 distinct kinds of the cubic Hénon maps have totally different bifurcations scenarios. Also bifurcations of related 4-periodic orbits are discussed. This is a joint work with S. Gonchenko, I. Ovsyannikov and A. Vieiro.

On bifurcations of two-dimensional diffeomorphisms with a homoclinic tangency to a nonhyperbolic saddle fixed point

O. Gordeeva, V. Lukyanov

Lobachevsky State University of Nizhny Novgorod

olga.gordeeva@inbox.ru, viluk@yandex.ru

We consider main bifurcations of two-dimensional diffeomorphisms having a quadratic homoclinic tangency to a nonhyperbolic fixed point of saddle type. Bifurcations of single-round periodic orbits from a small fixed neighborhood of the homoclinic orbit were studied.

Discrete Shilnikov attractor and chaos in the system of five identical globally coupled phase oscillators

Evgenij Grines[1], Alexey Kazakov[1,2]

[1] *National Research University Lobachevsky State University of Nizhny Novgorod*

[1,2] *National Research University High School of Economics*

grineseugene@mail.ru, kazakovdz@yandex.ru

In this talk we discuss the scenario of chaos emergence in the system of five identical globally coupled phase oscillators with Hansel-Mato-Meunier coupling. The system in consideration is an example of a system of identical phase oscillators which exhibits chaotic behaviour for certain values of parameters (see [1]). Our studies show that a birth of some of chaotic attractors in this system might be explained via the scenario of discrete Shilnikov attractor formation.

[1] Ashwin P, Orosz G, Wordsworth J, Townley S., Dynamics on networks of cluster states for globally coupled phase oscillators, 2007

On topological classification of structurally stable systems with regular and chaotic dynamics

V. Grines

vgrines@yandex.ru

Dynamics of a System of Two Differential Equations with Finite Nonlinear Feedback

Kashchenko A. A.

Yaroslavl State University

a.kashchenko@uniyar.ac.ru

Consider dynamics of a system of two coupled oscillators

$$\begin{aligned} \dot{u_1} + u_1 &= \lambda F(u_1(t-T)) + \gamma(u_2 - u_1), \\ \dot{u_2} + u_2 &= \lambda F(u_2(t-T)) + \gamma(u_1 - u_2). \end{aligned} \tag{1}$$

Here coupling parameter γ and delay time T are positive. Feedback function $F(u)$ is finite:

$$F(u) = \begin{cases} f(u), & |u| < p, \\ 0, & |u| \geq p, \end{cases}$$

where p is some positive constant. Function $f(u)$ is piecewise smooth, $f(u) < a < 0$ if $-p < u < 0$, $f(0) = 0$, and $f(u) > b > 0$ if $0 < u < p$. The main assumption is that parameter λ is sufficiently large ($\lambda \gg 1$).

The study is based on the use of a special method of large parameter. In phase space $C_{[-T,0]}(R^2)$ of system (1) a special set $S(x)$ dependent on parameter x is chosen. Then asymptotics of all solutions (1) with initial conditions from set $S(x)$ is studied at $\lambda \to \infty$. It is proved that after a certain period of time all considered solutions fall into the set $S(\bar{x})$, where for $\bar{x}$ one-dimensional mapping $\bar{x} = \phi(x)$ is obtained with accuracy $o(1)$ at $\lambda \to \infty$. Dynamics of this mapping define dynamics of initial system (1) principally.

It is shown that dynamics of (1) depends on coupling coefficient γ crucially. If γ is of order 1 then homogeneous solution ($u_1 \equiv u_2$) is stable. If $\gamma = const(\ln \lambda)^{-1}$ then more complex regimes arise, which are described by the mapping $\bar{x} = \phi(x)$. If γ is of order $\lambda^{-\alpha}(\ln \lambda)^{-1}$, where $0 < \alpha \leq 1/2$, it is proved that several inhomogeneous periodic regimes co-exist.

In all considered cases asymptotics of these relaxation periodic solutions of initial problem (1) was built.

This work was supported by State assignment, work 1.6074.2017/П220.

Local dynamics of non-linear equation with two large delays

Kashchenko I.

Yaroslavl State University

iliyask@uniyar.ac.ru

Consider equation with two large delays

$$\dot{x} + x = ax(t-T) + bx(t-T_1) + f(x, x(t-T), x(t-T_1)), \quad T_1 > T > 0.$$

Here a and b are some constants, $f(x, y, z)$ is nonlinear function. The main assumption is that delay times T and T_1 are sufficiently large and proportional each other:

$$T = \varepsilon^{-1}, \quad T_1 = T(k_0 + \varepsilon^{\alpha} k_1), \quad 0 < \varepsilon \ll 1, \quad k_0 \geq 1, \quad \alpha > 0.$$

The problem to research is to determine the behaviour of solutions in some small (but independent of ε) neighbourhood of zero equilibrium state and built the asymptotic for stable solutions.

It will be shown that algebraic properties of k_0 is very important. Results are different for rational and irrational k_0. Also, even the results on the stability of zero are significantly influenced by the values of α. So cases $\alpha < 1$, $\alpha = 1$ and $\alpha > 1$ will be studied separately.

It's proved that all critical cases have infinite dimension. For rational k_0 further investigation [1] was performed using the method of quasinormal forms. The main idea of this method is to construct a special substitution by means of which the initial equation reduces to a problem that does not contain small parameters (or depends on them regularly). This problem (the quasinormal form), unlike the initial equation, can easily be investigated numerically.

In all cases quasinormal forms are given and explicit formulas that connect the solutions of the original problem and the quasinormal form are presented. It turned out that normalized problems are nonlinear equations of parabolic type. In the case $\alpha > 1$, the space variable in these equations is one-dimensional, and for $\alpha \leq 1$, it is two-dimensional.

An important fact is the presence in quasinormal forms of arbitrary parameters (absent in the original equation). When these parameters change, we get another normalized problems, another their solutions and another solutions of the original equation. This indicates the presence of the phenomenon of multistability in the equation with two large delays.

1. Kashchenko I.S. Normalization of a system with two large delays // IJBC **24**:8 (2014) 1440021.

Families of normalized equations in the problem of dislocations in a solid

Kashchenko S.

Yaroslavl State University

kasch@uniyar.ac.ru

Consider a classical nonlinear differential equation with deviations of the space variable

$$m\ddot{y_n} + \alpha \sin y_n = \beta(y_{n+1} - 2y_n + y_{n-1}) \ (n = 1, \ldots, N). \tag{10}$$

Here, m, α and β are positive coefficients, and for $y_n = y_n(t, x_n)$, for extreme values of n, the conditions that are characteristic either of the periodic boundary value problem $y_{N+1} = y_1, y_0 = y_N$; or for the Dirichlet problem: $y_{N+1} = y_0 = 0$; or for the Neumann problem: $y_{N+1} = y_N, y_0 = y_1$. The points x_n are in the segment $[0, 2\pi]$ and $x_{n+1} = x_n + \varepsilon$, where $\varepsilon = 2\pi N^{-l}$. It is assumed that $N \gg 1$ or $0 < \varepsilon \ll 1$.

Passing to the continuous mass distribution, we obtain from (10) (after obvious renormalizations and the replacement of $\alpha \sin y$ by a more general function $f(y)$)

$$\ddot{y} + f(y) = y(t, x + \varepsilon) - 2y + y(t, x - \varepsilon). \tag{11}$$

The problem to research is to study under condition $\varepsilon \ll 1$ the behaviour of all the solutions of the boundary value problem (11) with initial conditions from some neighborhood of the zero equilibrium state of a sufficiently small (and independent of ε). To do this, we construct multiparameter families of non-linear systems of equations of a special form that play the role of normal forms. In particular, systems of nonlinear Schrodinger-type equations are given in a two-dimensional spatial domain.

Strange attractors and mixed dynamics in a problem on two vortexes

Alexey Kazakov[1,2]

[1] *National Research University High School of Economics*

[2] *Lobachevsky State University of Nizhny Novgorod*

kazakovdz@yandex.ru

In this talk, we consider a reversible system of equations describing the motion of two point vortices in a shear flow which has a constant uniform vorticity and which is perturbed by an acoustic wave. It is well known that without the influence of an acoustic wave, the system describing the motion of the vortices is integrable. But with the addition of an acoustic wave the system becomes nonintegrable, various (both regular and chaotic) attractors arise in it. We show that conservative behavior here is destroyed through symmetry breaking bifurcations due to which stable, completely unstable and symmetrical saddle orbits are born from symmetrical elliptic points (which lie on the line of fixed points of the involution). With further increase of the amplitude of an acoustic wave strange attractors and repellers are born from stable and completely unstable points due to cascades of period doubling bifurcations. We show that for some values of parameters these strange attractors and repellers may have a non-empty intersection. Moreover we research the homoclinic bifurcations leading to the intersection of strange attractors and strange repellers.

The work was supported by RSF grant 17-11-01041.

On spiral chaos of 3D flows

Alexey Kazakov[1,2], Alexander Korotkov[2]

[1] *National Research University High School of Economics*

[2] *National Research University Lobachevsky State University of Nizhny Novgorod*

kazakovdz@yandex.ru, koral81@bk.ru

In this work we present some results of the investigation of spiral chaos in 3-dimensional oscillator system and 3-dimensional generalized Lotka-Volterra system that were investigated in Arneodo-Coullet-Tresser papers [1,2] and in Rossler system [3]. It is shown that in these systems the chaotic attractors appear due to Shilnikov's scenario [4]: with changing a parameter a stable limiting cycle and a saddle-focus equilibrium are born from stable equilibrium in the. Then the unstable invariant manifold of saddle-focus winds on the stable limit cycle and forms a whirlpool. For some value of the parameter the unstable invariant manifold touches one-dimensional stable invariant manifold and forms homoclinic trajectory to saddle-focus. If in this case the limiting cycle loses stability (for example due to cascade of period-doubling bifurcations) and saddle value of the saddle-focus is negative then a strange attractor based on the loop to the saddle-focus appears. Using diagrams of change of topology which contains "bifurcation" curves corresponding to the appearence of a new extremum in 1-dimensional map (that can be built in such systems) we demonstrate that the so-caled periodicity hubs which are clearly visible on the diagrams of maixmal Lyapunov exponent arise when a new maximum in 1D map appear and its image goes to the saddle-focus.

Acknowledgments. The work was supported by RSF grant 14-12-00811.

[1] Arneodo A., Coullet P., Tresser C. Occurence of strange attractors in three-dimensional Volterra equations. Physics Letters A., 79(4), 1980.

[2] Arneodo A., Coullet P., Tresser C. Possible new strange attractors with spiral structure. Commun. Math. Phys, 79, 1981.

[3] Rossler O. E. An equation for continuous chaos. Physics Letters A., 57(5), 1976.

[4] L. P. Shilnikov. The theory of bifurcations and turbulence. Selecta Math. Sovietica, 10 (1991), 43-53.

Liquid crystal defects in the Landau-de Gennes theory-beyond the one-constant approximation

G. Kitavtsev

Bristol Univ., UK

georgy.kitavtsev@bristol.ac.uk

In this talk the two-dimensional Landau-de Gennes energy with several elastic constants, subject to general k-radial symmetric boundary conditions, will be analysed. It will be shown that for generic elastic constants the critical points consistent with the symmetry of the boundary conditions exist only in the case k=2. In this case one can identify three types of radial profiles: with two, three of full five components. Next, numerical results on domains of existence and stability of these radial solutions as well as of certain non-radial ones, so called two 1/2-defects solutions, will be collectively presented and discussed on the corresponding bifurcation diagrams in two cases: the usual case when the bulk energy vanishes on a uniaxial set of co-dimension 3, and degenerate one when it vanishes on a biaxial set of co-dimension 1. In the final part of the talk different paths for emergency of non-radially symmetric solutions and their corresponding analytical structure in 2D as well as 3D cases will be discussed. These results is a joint work with Jonathan Robbins, Valery Slastikov and Arghir Zarnescu.

Hard and soft oscillation excitation in memristor-based oscillator with a line of equilibria

I. Korneev

Saratov State University, Russia

Using flow saddle charts for searching homoclinic attractors

A. Kozlov

National Research University Lobachevsky State University of Nizhny Novgorod

kozzzloff@list.ru

This work focuses on the problem of existence of homoclinic attractors in three-dimensional flows of the following type $\dot{x} = y, \dot{y} = z, \dot{z} = Ax + By + Cz + g(x, y), g(0, 0) = g'_x(0, 0) = g'_y(0, 0) = 0$. Homoclinic attractors are the strange attractors which contain only one (saddle) equilibrium point. The type of such attractors is defined by eigenvalues of the equilibrium point, which depend only on parameters A, B, and C. A method of saddle charts (two-parameter diagram in which regions with different eigenvalues are drawn with different colors) together with methods of charts of maximal Lyapunov exponent and charts of the distance between an attractor and a saddle point (to verify that a saddle point belongs to the attractor) are used for searching and classifying of homoclinic attractors. Using these methods it was found attractors of spiral and also Shilnikov types

Investigation of an asymmetric pendulum type equation

Kostromina O.S.

Lobachevsky State University of Nizhny Novgorod

os.kostromina@yandex.ru

We study dynamics and bifurcations of an asymmetric pendulum type equation close to an integrable one. For an autonomous equation we construct partition of the parameters plane into domains with different phase portrait topology. We give exact estimates for the number of limit cycles of the equation. For a nonautonomous equation we solve the problem of the existence of homoclinic structures in the neighborhood of unperturbed separatrices. We construct bifurcation diagram for the Poincarщ map on the parameters plane separating domains

of existence of different homoclinic structures. The results obtained are illustrated by numerical computations.

This work was partially supported by RFBR, No 16-01-00364.

Clustering and chimeras in the model of the spatial dynamics of age-structured populations

Kulakov M.P., Frisman E.Ya.

Institute for Complex Analysis of Regional Problems FEB RAS, Birobidzhan, Russia

k_matvey@mail.ru, frisman@mail.ru

The abstract is devoted to problem of clustering in ring of nonlocal coupled two-dimensional nonlinear maps. Single system is a model of two-age population and coupling is a migration of adult individual. In this case, the model equations have the following form:

$$\begin{cases} x_{n+1}^{(i)} = a y_n^{(i)} \exp\left(-\rho x_n^{(i)} - y_n^{(i)}\right) \\ y_{n+1}^{(i)} = s x_n^{(i)} + v y_n^{(i)} + \frac{mv}{2S} \sum\limits_{j=i-S}^{i+S} \left(y_n^{(j)} - y_n^{(i)}\right) \end{cases}, (i = 1, 2, \ldots, N) \qquad (1)$$

where $x_n^{(i)}$ and $y_n^{(i)}$ is the number of junior and senior individuals, parameters $a > 0$, $v > 0$ and $s > 0$ characterize the growth of a single population, N is the number of population sites (oscillators) in the ring, S and m is coupling radius and strength. The ring area defined by index j equal to periodic $j \bmod N$.

The case of $N = 2$ and $S = 1$ was investigated in [1]. It was shown that system 1 demonstrates the multistability of two types. The first one is the synchronization of the coupled partial systems, corresponding to phase multistability. The second one is the multistability of the partial system at $m = 0$, which consists in the fact that simultaneously there are the stable fixed point and stable periodic point of period 3 and regimes after their bifurcations. It was shown the coupling between single oscillators leads to a combination of these types of multistability. At first these modes can be synchronous and asynchronous. Secondly asynchronous or partial synchronization modes are realized in many different ways and they differ not only in the phases (phase multistability) but the amplitudes and the even periods.

This report focuses on the results of the system (1) study at $N > 2$ and nonlocal coupling ($S > 1$). Regimes and conditions for the formation of clusters in this model are numerically investigated with variation coupling radius S and strength m.

It was found the multistability of dynamics of a local two-aged population (single oscillator at $m = 1$) is capable of leading the fact that each cluster can oscillate with

Research, USA [2], Mayo Clinic, USA [3], and other research groups. Industrial companies are also very interested development of new technologies of treatment using bioelectronic approach [4, 5]. Increasing number of papers in this field and signed big contracts show significant interest in development of revolutionary new strategy in treatment of different diseases. According to the newest studies published in [6], bioelectronic approach may be successfully used not only in treatment of neural diseases, but also in treatment of cardiovascular, inflammatory, metabolic and endocrine diseases. These facts are confirmed by animal studies and first clinical studies. The nervous system is the main controller of all intrinsic processes in body, from emotions and thinking to digestion and motor activity [7]. For these reasons, there is an increasing interest in electrical couplings in neural system and their role in generation of different regimes of neural activity, in mechanisms of their appearance and destruction.

However new medical technologies development and their implementation in real therapy demands much deeper understanding of peripheral neural system and its role in regulation of processes in human body. We study influence of strength and topology of electrical couplings on regimes of neuron-like activity in phenomenological model of neuronal ensembles with chemical (synaptic) and electrical couplings. Individual element in this case is modelled using Van der Pol equations. In previous studies, we studied ensembles of neuron-like elements with only chemical inhibitory couplings [8]. In this paper different dynamical regimes that can be observed in the case of chemical couplingsT parameters changing were studied. However, for more biological relevant results it is necessary to take in account the influence of coupling strength and topology of electrical couplings between elements as well as nonidentity of elements as it was described in various neurobiological literature [9]. We study the influence of strength and topology of electrical couplings on dynamics of ensemble of neuron-like nonidentical elements with chemical inhibitory couplings.

The model

$$\begin{cases} \ddot{x_j} - \mu[\lambda(x_j, \dot{x_j}) - x_j^2]\dot{x_j} + \omega_j^2 x_j + d(x_{j+1} - 2x_j + x_{j-1}) = 0, \\ j = 1, 2, 3. \end{cases} \tag{13}$$

Variable x_j phenomenologically describes changing in membrane potential of j-th element. Electrical couplings between elements are described by $d(x_{j+1} - 2x_j + x_{j-1})$, where parameter d is the strength of electrical coupling. Chemical (synaptic) inhibitory couplings are described using parameter λ

$$\lambda(x_j, \dot{x_j}) = 1 - g_1 F(\sqrt{x_{j+1}^2 + \dot{x}_{j+1}^2}) - g_2 F(\sqrt{x_{j-1}^2 + \dot{x}_{j-1}^2}),$$

where g_1 and g_2 are strength of inhibitory clockwise and anti-clockwise couplings respectively. Function $F(z)$ is an activation function with threshold value z_0 that phenomenologically describes the main principle of synaptic coupling:

$$F(z) = \frac{1}{1 + exp(-k * (z - z_0))}.$$

It is well known that in real experiments frequencies that are registered for different neurons and neuronal clusters are differ. We describe this fact by adding to the system (13) new parameter of frequency mismatch Δ, that can be set as $\Delta = \omega_2 - \omega_1 = \omega_3 - \omega_2$.

To study this system we built maps of the largest Lyapunov exponents on plane (Δ, d) and mark regions of different regimes that can be observed in the system on it. We show that adding electrical couplings and frequency mismatch leads to occurrence of new types of dynamics, including periodic, quasiperiodic and chaotic.

[1] Seo D., et al. Neuron. 2016. V. 91 (3). P. 529. [2] http://www.feinsteininstitute.org/programs-researchers/bioelectronic-medicine/ [3] http://www.mayo.edu/research/labs/neural-engineering/research-activities/bioelectronic-medicine [4] Electrocore LLC http://www.electrocore.com [5] GSK and Alphabet's Verily create $715M joint venture in bioelectronics, http://www.fiercebiotech.com/medical-devices/gsk-and-google-s-verily-create-715m-joint-venturebioelectronics, http://www.theverge.com/2016/8/1/12340454/gsk-google-bioelectronic-medicine-company [6] Birmingham K.,et al. Nature Reviews Drug Discovery. 2014. V. 13. P.399. [7] Afraimovich V.S., Zhigulin V.P., Rabinovich M.I. Chaos. 2004. V. 14(4). P. 1123. [8] Levanova T.A., Komarov M.A., Osipov G.V. EPJ ST. 2013. V.222(10) . P. 2417. [9] Nicholls J.G. et al. From Neuron to brain. 5th ed. Sinauer Associates, 2011.

On partially hyperbolic symplectic automorphisms of a 4-dimensional torus generated by integer unitary symplectic matricies

L.Lerman, K.Trifinov

Lobachevsky State University of Nizhny Novgorod

lermanl@mm.unn.ru

We study partially hyperbolic symplectic automorphisms of a 4-dimensional torus generated by integer unitary symplectic matricies. Before only hyperbolic torus maps were mainly studied, the investigations of partially hyperbolic automorphisms started rather recently and there are still many open questions. In this paper, we construct examples of partially hyperbolic symplectic maps on the four-dimensional torus with various dynamics. In particular, we construct suchmap on the torus which has transitive one-dimensional foliations generated by the eigendirections corresponding to eigenvalues lying on the unit circle. In order to construct such an example, the theory of irreducible polynomials with integer coefficients and some properties of invariant subspaces of the related integer unitary matrices are exploited.

Estimates on jumps of entropy for perturbations of generalized interval exchange transformations

M.I. Malkin

Lobachevsky State University of Nizhny Novgorod

malkin@unn.ru

We consider one-dimensional piecewise monotone discontinuous maps along with their one-dimensional and multidimensional perturbations by using the technique of kneading invariants and kneading series. The kneading technique was introduced first by J. Milnor and W. Thurston for continuous piecewise monotone one-dimensional maps and was applied before to maps with positive topological entropy. In the present talk we allow also a more complicated case in neighborhoods of a map with zero topological entropy. We show how to use the kneading techique for Lorenz maps with zero entropy and for generalized interval exchange transformations, i.e., at the border of the convergence disk for kneading series in the complex plane, in order to construct the invariant measures and thus, to construct semiconjugacy (being actually a conjugacy in the transitive case) with minimal model maps of unit slope, i.e., for rigid interval exchange transformations.

Concerning Lorenz maps, the following result holds.

Theorem 1 The function $f \to h_{top}(f)$ in the class of Lorenz maps with C^0-topology is continuous at f_0, except for the case when $h_{top}(f_0) = 0$ and the kneading invariants $K^+_{f_0}, K^-_{f_0}$ of f_0 are periodic with the same period; in the latter case, the jump of topological entropy is precisely $\frac{1}{p}\log 2$, where p is the common period of the kneading invariants. Moreover, for the class of Lorenz maps having zero one-sided derivatives at the discontinuity point and with C^1-topology, such an exceptional case is impossible, and thus, the topological entropy depends continuously on the map.

For the case of piecewise-monotone, piecewise-continuous maps, an exact estimate on possible jump of topological entropy with C^0-topology is given in the following theorem which extends a similar result by M. Misiurewich proven for continuous maps.

Theorem 2 Let a piecewise monotone map f_0 has points of discontinuity (or critical points) $\{c_1, \ldots, c_{n-1}\}$ and let $B(t) = (b_{i,j}(t))$ be the matrix function, with $b_{i,j}(t) = \sum_{i,j} t^{\ell_{i,j}}$, where $\ell_{i,j}$ is the lengh of path from c_i to c_j (if exists) and the sum is taken over all such paths. Then the maximal possible jump of the topological entropy for perturbations at f_0 with repect to C^0-topology is equal to

$$-\log(\text{minimal positive root of the polynomial} \quad det(E - B(t)))$$

We also discuss multidimensional perturbations of these results.

The work was done in collaboration with K.A. Safonov. It was partially supported by RFBR grant 15-01-03687-p.

Synchronization in multi-time scale multiplex networks

S. Makovkin, M. Ivanchenko, A. Zaikin, S. Jalan

Lobachevsky State University of Nizhni Novgorod

The nervous system in brain is made of neurons, specialized cells that can receive and transmit chemical or electrical signals, and glial cells which support, nourish, and protect the neurons. In this work we investigate impact of the glial cells activities on synchronizability of neural cells in multiplex networks framework. A multiplex network in which one layer represents interactions among the <glial> cells and the other layer represents those of <neural> cells is taken. Connections among the <glial> cells form a regular star like periodical structure in which each cell is connected to the four other neighbour cells whereas connections, among <neural> cells are represented by an Erdos-Renyi random network with average quantity connections is equal by four. Inter-layer links are such that each node in the <neural> layer is connected to its mirror in <glial> layer and all the four neighbours of the mirror node.

On the transitory FitzHugh-Hagumo model

A.D. Morozov, K.E. Morozov

Lobachevsky State University of Nizhni Novgorod

morozov@mm.unn.ru

For the autonomous FitzHugh-Nagumo system the splitting of a parameter plane to domains with different structure is established while is supposed fixed. The case of fast-slow motions is also considered. In the case of one equilibrium state the region with two limit cycles was found as well as in the case of three equilibrium states the region with three limit cycles was found. For the transitory model (is a time-dependent function only on a compact interval of time) the effect of non-autonomous part to establishing one or another mode is considered. The transition from one domain of the parameter plane to other is discussed.

Dynamical mechanisms of the excitability type of dopaminergic neuron and how it is influenced by the intrinsic and synaptic inputs

E.O. Morozova[1,3], D. Zakharov[2], A. Kuznetsov[3]
Justus Kromer[2], Lutz Schimansky-Geier[2]

[1] *Department of Physics, Indiana University Bloomington, USA*

[2] *Institute of Applied Physics, Russian Academy of Sciences, Nizhny Novgorod, Russia*

[3] *Department of Mathematical Sciences, IUPUI, Indianapolis, USA*

morozova.e.o@gmail.com

Intrinsic membrane properties of individual neurons play a central role in defining their spike generation mechanism, responses to various stimuli, synchronization properties and neural coding strategy. The type of neural excitability depends on the type of bifurcation from resting to spiking state. In this study we described the type of excitability of the dopamine (DA) neurons and how it is affected by the current composition. Determining the type of excitability helps us to predict the behavior of the DA neuron during application/blockade of different currents and better understand computations it performs in different input conditions. We used a one compartmental 2D model of the DA neuron that captures main features of the DA neuron firing. We applied a phase plane analysis and bifurcation theory to describe the excitability type of the DA neuron and conditions for switching from one type of excitability to another. We then attempted to link biophysical properties of the DA neurons with the computations they perform

in the brain and possibly different behavioral states. Particularly, we investigated the impact of bifurcation mechanisms of DA neurons' excitability on the collective activity of the neurons and, as a result, produced DA levels. We studied how spike initiation dynamics affects computational properties of individual DA neurons, for example, encoding reward-related stimulus intensity, which is one of the most prominent functions of the DA neurons. We showed that spike initiation dynamics and, accordingly, operation mode of the DA neuron can be modulated by changes in certain intrinsic and synaptic currents, which could be caused, for example, by various drugs of abuse.

Emergent stochastic oscillations and information processing in tree networks of excitable elements

Alexander Neiman [1], Ali Khaledi Nasab [1],
Justus Kromer[2], Lutz Schimansky-Geier[2]

[1] *Ohio University*

[2] *Humboldt University at Berlin*

We study the stochastic dynamics of strongly-coupled excitable elements on tree networks. The peripheral nodes receive independent random inputs which may induce large spiking events propagating through the branches of the tree and leading to global coherent oscillations in the network. This scenario may be relevant to action potential generation in certain sensory neurons, which possess myelinated distal dendritic tree-like arbors with excitable nodes of Ranvier at peripheral and branching nodes, and may exhibit noisy periodic sequences of action potentials. A biophysical model of distal branches of a sensory neuron in which nodes of Ranvier are coupled by myelinated cable segments is used along with a generic model of networked stochastic active rotators. We focus on the spiking statistics of the central node, which fires in response to independent noisy inputs at peripheral nodes. We show that, in the strong coupling regime, relevant to myelinated dendritic trees, the spike train statistics can be predicted from an isolated excitable element with rescaled parameters according to the network topology. Furthermore, we show that by varying the network topology the spike train statistics of the central node can be tuned to have a certain firing rate and variability, or to allow for an optimal discrimination of inputs applied at the peripheral nodes.

On Hoffbauer conjecture

C. Olszowiec

Imperial College, London

c.olszowiec14@imperial.ac.uk

Global bifurcations and discrete Lorenz attractors

Ovsyannikov I.I.

Universität Bremen, Germany

Ivan.I.Ovsyannikov@gmail.com

A presence of non-transversal homoclinic or heteroclinic orbits (tangencies) in a dynamical system is regarded as a universal criterion of existence of a complex dynamics. However, it does not immediately lead to the emergence of genuine strange attractors (i.e. those preserving their "strangeness" under small perturbations) such as the Lorenz attractors. In the talk the list of three-dimensional diffeomorphisms with quadratic homoclinic and heteroclinic tangencies is presented in which discrete Lorenz attractors are born in bifurcations. For some of them a stronger result was also proved: in any neighbourhood there exist residual sets in which systems possess a countable number of coexisting discrete Lorenz attractors. Note that the bifurcations under consideration can be of codimension three, two and even one. In order to get Lorenz attractors one needs to have the effective dimension of the problem to be not less than three. For 3D diffeomorphisms this means that there should be no global contraction/expansion and no global center manifolds. To fulfill the first condition the Jacobian at the saddle fixed point should be close to 1 in the homoclinic case, and in the heteroclinic case it is enough to have the so-called contracting-expanding configuration, when the Jacobian at one saddle is less than one and greater than one at another saddle. The following conditions prevent the appearance of center manifolds: 1. At least one of the saddle fixed point is a saddle-focus; 2. All the fixed points are saddles but one of the homoclinic/heteroclinic orbits is non-simple; 3. A saddle is resonant (two stable eigenvalues either coincide or have opposite signs).

Energy function for dynamical systems

O. V. Pochinka

National Research University Higher School of Economics, Nizhny Novgorod

olga-pochinka@yandex.ru

Let M be a smooth compact orientable n-manifold. *Lyapunov function* of a dynamical system (flow or cascade) defined on M is a continuous function $\varphi : M \to \mathbb{R}$ that is constant on each chain component of the system and decreases along its orbits outside the chain recurrent set. By the results of Ch. Conley [1], such a function exists for any dynamical system, and the fact of existence is called "Fundamental theorem of dynamical systems". It should be noted that Ch. Conley himself additionally required that the image of a chain recurrent set in view of φ be nowhere dense on a line, and the values тАЛтАЛof φ on different components of a chain recurrent set are distinct and call a function *the complete Lyapunov function.* Ch. Conley called by critical numbers of the function φ the numbers belonging to the image of the chain-recurrent set. However, for a smooth function, its critical value is the image of the critical point (the point at which the gradient of the function vanishes), which, generally speaking, does not need to belong to a chain-recurrent set. In connection with this, in addition to the Lyapunov function, in the smooth category we use the concept of *of the energy function*, that is, a smooth Lyapunov function whose set of critical points coincides with the chain-recurrent set of the system.

The first results on the construction of the energy function are due to S. Smale [2], who in 1961 proved the existence of the Morse energy function for gradient-like flows. K. Meyer [3] in 1968 generalized this result by constructing the Morse-Bott energy function for an arbitrary Morse-Smale flow.

As J. Franks noted in [4] in 1985, the application of Wilson's results [5] to Conley's construction gives the existence of an energy function for any smooth flow with a hyperbolic chain-recurrent set. Then, using the suspension, we can construct a smooth Lyapunov function for any diffeomorphism with the hyperbolic chain-recurrent set. But, the function constructed in this way can have critical points that are not chain recurrent and, hence, the Lyapunov function is not energy. The question arises as to which discrete dynamical systems admit energy functions. The first results in this direction were obtained by D. Pixton in 1977, in his paper [6], he proved the existence of the Morse energy function for any Morse-Smale diffeomorphism on the surface. In 2012, T. Mitryakova, O. Pochinka, A. Shishenkova generalized the result of Picston to Ω-stable 2-diffeomorphisms with a finite non-wandering set, the Morse energy function for such diffeomorphisms was constructed in the paper [7]. In the same paper [6], D. Pixton constructed a Morse-Smale diffeomorphism on a three-dimensional sphere that does not have the Morse energy function. Necessary and sufficient conditions for the existence of the Morse energy function in three-dimensional Morse-Smale diffeomorphisms are found in the papers [8], [9] and [10] by V. Grines, F. Laudenbach, O. Pochinka. In addition, by the same authors in the paper [11] it is proved that in the example of a Pixton the minimum number of critical points of the Lyapunov function other than the periodic points of the cascade is two.

It follows from the above that not all diffeomorphisms, even with regular dynamics, have an energy function. The more surprising is the fact that some discrete dynamical systems with chaotic behavior have the energy function, proved by V. Grines, M. Noskova, O. Pochinka in the works [12], [13], [14] for some classes of Ω-stable 2- and 3-diffeomorphisms with nontrivial basis sets of codimension one. Technically, the construction of such a function is based on the smoothing procedure for continuous map.

Despite the considerable progress in the construction of energy functions for discrete dynamical systems, many open questions remain. In particular, there is no algorithm for constructing such a function for arbitrary Ω-stable diffeomorphisms of surfaces, although, apparently, such a function exists. Moreover, there is no technique for constructing energy functions on manifolds of large dimension even in the class of regular dynamical systems.

References

[1] Conley C. Isolated Invariant Sets and Morse Index. CBMS Regional Conference Series in Math. — 1978. V. 38.

[2] Smale S. On gradient dynamical systems. Annals Math. —1961. —V. 74. —P.199–206.

[3] Meyer K. R. Energy functions for Morse-Smale systems. Amer. J. Math. —1968. —V. 90. — P. 1031–1040.

[4] Franks J. Nonsingular Smale Flow on S^3. Topology. —1985. —V. 24, No 3. — P. 265–282.

[5] Wilson W. Smoothing derivatives of functions and applications. Trans. Amer. Math. Soc. —1969. — V. 139. —P. 413–428.

[6] Pixton D. Wild unstable manifolds. Topology. —1977. —V. 16. —P. 167–172.

[7] Mitryakova T., Pochinka O., Shishenkova A. Energy function for diffeomorphisms of surfaces with finite hyperbolic chain recurrent set. Journal of the Middle Volga Mathematical Society. — 2012. — V. 14, No 1. — P. 98–107.

[8] Grines V., Laudenbach F., Pochinka O. Self-indexing enrgy function for Morse-Smale diffeomorphisms on 3-manifolds. Moscow Math. Journal. — 2009. — V. 9, No 4. — P. 801–821.

[9] Grines V., Laudenbach F., Pochinka O. Dynamically ordered energy function for Morse-Smale diffeomorphisms on 3-manifolds. Proceedings of the Steklov Institute Of Mathematics. — 2012. — V. 278, No 1. — P. 27–40.

[10] Grines V., Medvedev T., Pochinka O. Dynamical Systems on 2- and 3-Manifolds. Springer International Publishing Switzerland. 2016. — 364 P.

[11] Grines V., Laudenbach F., Pochinka O. Quasi-energy function for diffeomorphisms with wild separatrices. Mathematical notes. — 2009. — V. 86, No 2. — P. 175–183.

[12] Grines V., Noskova M., Pochinka O. Construction of the energy function for A-diffeomorphisms with a two-dimensional non-wandering set on 3-manifolds. Proceedings of the Middle Volga Mathematical Society. — 2015. — V. 17, No 3. — P. 12–17.

[13] Grines V., Noskova M., Pochinka O. Construction of the energy function for three-dimensional cascades with a two-dimensional expanding attractor. Proceedings of the Moscow Mathematical Society. — 2015. — V. 76, No 2. — P. 271–286.

[14] Grines V., Zhuzhoma E., Pochinka O. Rough diffeomorphisms with Basic sets of codimension one. Mathematical science. — 2015. — V. 57. — P. 5–30.

Oscillations in antiphase in a system of two non-linearly coupled relaxation oscillators

M. M. Preobrazhenskaia

Yaroslavl State University, Scientific Center in Chernogolovka RAS

rita.preo@gmail.com

Let us consider the system of non-linear differential-difference equations (see [1,2])

$$\begin{aligned} \dot{u}_1 &= [\lambda f(u_1(t-1)) + b\, g(u_2(t-h)) \ln(u_*/u_1)]u_1, \\ \dot{u}_2 &= [\lambda f(u_2(t-1)) + b\, g(u_1(t-h)) \ln(u_*/u_2)]u_2, \end{aligned} \tag{1}$$

which is used to model the association of two neurons with a synaptic connection. Here $h > 0$ characterizes the delay in the connection chain, $u_1(t), u_2(t) > 0$ are the normalized membrane neurons potentials, the parameter $\lambda \gg 1$ characterizes the electrical processes rate in the system, $b = \text{const} > 0$, $u_* = \exp(c\lambda)$ is the threshold value for control interaction, $c = \text{const} \in R$, the terms $bg(u_{j-1})\ln(u_*/u_j)u_j$ model synaptic interaction. The functions $f(u),\ g(u) \in C^2(R_+)$, where $R_+ = \{u \in R : u \geq 0\}$, and satisfy the following conditions:

$$\begin{aligned} &f(0) = 1; \quad f(u) + a,\ uf'(u),\ u^2 f''(u) = O(u^{-1}) \text{ as } u \to +\infty,\ a = \text{const} > 0; \\ &\forall u > 0\ g(u) > 0,\ g(0) = 0; \quad g(u) - 1,\ ug'(u),\ u^2 g''(u) = O(u^{-1}) \text{ as } u \to +\infty. \end{aligned} \tag{2}$$

For any natural n we find a periodic solution of the system (1) containing n asymptotically high bursts on the period.

Note, that the system (1) has a synchronous solution $u_1 \equiv u_2$.

Let us show that it has another solution in the form

$$u_1(t) = u(t), \quad u_2(t) = u(t + \Delta), \tag{3}$$

where Δ is a positive constant and $u(t)$ is the periodic solution of equation

$$\dot{u} = [\lambda f(u(t-1)) + b\, g(u(t + \Delta - h)) \ln(u_*/u)]u. \tag{4}$$

After replacing $u = \exp(\lambda x)$, $\lambda = 1/\varepsilon$, $\varepsilon \ll 1$, the equation (4) is modified to

$$\dot{x} = f(\exp(x(t-1)/\varepsilon)) + b\,(c - x) g(\exp(x(t + \Delta - h)/\varepsilon)), \varepsilon). \tag{5}$$

Using (2) we obtain that it has limited equation

$$\dot{x} = R(x(t-1)) + b\,(c - x) H(x(t + \Delta - h)), \tag{6}$$

where

$$R(x) \stackrel{\text{def}}{=} \begin{cases} 1 \text{ as x<0}, \\ -a \text{ as x>0}, \end{cases} \qquad H(x) \stackrel{\text{def}}{=} \begin{cases} 0 \text{ as x<0}, \\ 1 \text{ as x>0}. \end{cases}$$

Denote the solution of the relay equation (6) by $x_*(t)$, and its period by T_*. The constant Δ satisfy the matching condition $\Delta = T_*/2$.

Let's formulate the

Theorem. *The parameters a, b, c, h, Δ exist such that for any natural n and small enough $\varepsilon > 0$ the equation (5) has an orbital exponential stable cycle $x_*(t, \varepsilon)$ with period $T_*(\varepsilon)$, at that*

$$\lim_{\varepsilon \to 0} \max_{0 \leq t \leq T_*(\varepsilon)} |x_*(t, \varepsilon) - x_*(t)| = 0, \quad \lim_{\varepsilon \to 0} T_*(\varepsilon) = T_*,$$

and period of the function $x_*(t,\varepsilon) > 0$ has n segments where it is positive.

Proved theorem allows us to justify the existence and stability of a periodic solution of the form (3) with n asymptotically high bursts on the period for the system (1).

Acknowledgments: This work was partly supported by the Russian Science Foundation (project nos. No 14-21-00158).

References

[1] Glyzin S. D., Kolesov A. Y., Rozov N. K. On a method for mathematical modeling of chemical synapses // Differential Equations. 2013. V. 49, No 10. P. 1227–1244.

[2] Preobrazhenskaia M. M. Relaxation cycles in a model of synaptically interacting oscillators // Modeling and Analysis of Information Systems. 2017. V. 24, No 2. P. 186–204.

Symbolic computations to untangle homoclinic chaos and its origin, order, and organization

K. Pusuluri

pusuluri.krishna@gmail.com

On some exact solutions of Yang-Mills equations

A. E. Rassadin

Nizhny Novgorod mathematical society

brat_ras@list.ru

Theory of non-abelian gauge fields which has been arisen after the pioneer work [1] of Yang and Mills is an example of nonlinear theory of field. When instanton has been discovered in article [2] it has been clear that there are untrivial connections between exact solutions of Yang-Mills equations and topology. But at the same time papers [3, 4] demonstrating interrelations between Yang-Mills equations and theory of dynamical systems have been appeared too. In the review presented the detailed derivation of Yang-Mills equations from the Lagrangian density invariant under gauge group SU(2) will be done. After that a number of examples showing how to reduce nonlinear system of twelve second-order partial differential equations for potentials of the Yang-Mills field to some systems of ordinary differential equations which can demonstrate chaotic behaviour. The main purpose of this survey is to attract attention of young representatives of L. P. Shilnikov's school for nonlinear dynamics to investigations in the abovedescribed and actively developing branch of modern science [5].

References:

1. Yang C.N., Mills R.L. Conservation of isotopic spin and isotopic gauge invariance // Physical Review. 1954. V. 96. P. 191 - 195.

2. Belavin A.A., Polyakov A.M., Schwartz A.S., Tyupkin Yu.S. Pseudoparticle solutions of the Yang-Mills equations // Physics Letters B. 1975. V. 59. P. 85 - 87.

3. Matinyan S.G., Savvidy G.K. and Ter-Arutyunyan-Savvidy N.G. Classical Yang-Mills mechanics. Nonlinear color oscillations // Sov. Phys. JETP, 80, 830-838, 1981; JETP Lett. 34, 590, 1981.

4. Nicolaevskii E.S., Shchur L.N. The nonintegrability of classical Yang-Mills equations // Zh. Eksp. Teor. Fis., 85, 3-13, 1983.

5. http://www.claymath.org/millennium-problems/yang-mills-and-mass-gap .

Bispectra of dynamical variables of Rikitake system

Kostromina O.S.[1], Potapov A.A.[2], Rakut I.V.[1], Rassadin A.E.[3]

[1] *Lobachevsky State University of Nizhny Novgorod, Russia*

[2] *Kotel'nikov Institute of Radio Engineering and Electronics of RAS*

[3] *Nizhny Novgorod Mathematical Society*

os.kostromina@yandex.ru, potapov@cplire.ru, brat_ ras@list.ru

Up to the report [1] the Rikitake model [2] was considered as purely mathematical model. But in [1] it was suggested to make physical model of this two-disk dynamo system in order to obtain real generator of chaotic output voltage. In this report triple correlation functions for the Rikitake model observables and their bispectra [3] both in regular and in chaotic regime have been presented. We claim that higher-order statistic analysis of time series is true alternative to conventional in theory of dynamical chaos spectral-correlation approach because bispectrum can be used to detect the interactions between Fourier components of observables and to reveal information about phase coupling of observables [3].

References:

1. Potapov V.I., Rakut I.V., Rassadin A.E. Generalized Rikitake systems and their applications // International conference 'Shilnikov WorkShop 2014', dedicated to the 80th birthday of Professor Leonid Pavlovich Shilnikov (Nizhny Novgorod, Russia, December 17-19, 2014). Book of abstracts. P. 36-37.

2. Cook A.E., Roberts P.H. The Rikitake two-disk Dynamo System // Proc. of Cambridge Phylosophycal Society. 1970. V. 68. P. 547-569.

3. Malakhov A.N. Cumulant analysis of random non-gaussian processes and their transformation. Moscow, Sov. Radio, 1978.

Radiation of chaotic electromagnetic waves by the ferromagnetic or ferroelectric triple linkage of Thurston and Weeks

Potapov A.A.[1], Rakut I.V.[2], Rassadin A.E.[3], Sinitsky I.O.[2], Tronov A.A.[3]

[1] *Kotel'nikov Institute of Radio Engineering and Electronics of RAS*

[2] *Lobachevsky State University of Nizhny Novgorod, Russia*

[3] *Nizhny Novgorod Mathematical Society*

potapov@cplire.ru, rakut@nirfi.unn.ru, brat_ras@list.ru

In the report [1] construction of generator of chaotic electrical oscillations of new mechanical and electrodynamical type has been offered. One can say that this device transforms mixed dynamics introduced in [2] into electromagnetic field. It is well known that the triple linkage hinge mechanism of Thurston - Weeks possesses by chaotic dynamics too [3]. Therefore if one makes hinges from ferroelectric or ferromagnetic then temporal dynamics of electric or magnetic moments of hinges will follow the chaotic temporal dynamics of its angular variables. It means that the electromagnetic field excited in the near zone will be chaotic. So if one surrounds the triple linkage hinge mechanism of Thurston - Weeks by the metal box with a low-frequency connector then chaotic voltage oscillations there will arise. The device described do not include semiconducting elements at all that is why it will be stable under the action of extremal conditions of exploitation such as heavy elementary particles or high temperature.

References:

1. Gonchenko A.S., Rakut I.V., Rassadin A.E., Sinitsky I.O., Tronov A.A. The ferromagnetic or ferroelectric Suslov top as generator of chaotic electric oscillations // International conference 'Shilnikov WorkShop-2016', dedicated to the memory of Leonid Pavlovich Shilnikov (1934-2011) (Nizhny Novgorod, Russia, December 16-17, 2016). Book of abstracts. P. 20-21.

2. Gonchenko S.V. Elements of contemporary theory of dynamical chaos // International Conference-School "Dynamics, Bifurcations and Chaos 2016 (DBC III)" (Nizhny Novgorod, Russia, July 18-22, 2016). Book of abstracts. P. 14.

3. Kuznetsov S.P. Hyperbolic chaos in self-oscillating systems based on mechanical triple linkage: Testing absence of tangencies of stable and unstable manifolds for phase trajectories // arXiv: 1510.03593v1 [nlin.CD].

Scenarios of transition to chaos in the model of population with age and sex structures

Revutskaya O.L.[1], Kulakov M.P.[1], Neverova G.P.[2], Frisman E.Ya.[1]

[1] *Institute for Complex Analysis of Regional Problems FEB RAS, Birobidzhan, Russia*

[2] *Insititute for Automation and Control Processes FEB RAS, Vladivostok, Russia*

oksana-rev@mail.ru, k_ matvey@mail.ru, galina.nev@gmail.com, frisman@mail.ru

This paper investigates scenarios of transition to chaos in the model of population with age and sex structures and density-dependent regulation of juveniles' survival. We suggest mathematical model that enables the simultaneous observation of the formation of both age and sex structures and the explicit consideration of the asymmetry of the effects of females and males on population processes. We develop this model for situations wherein a population may be regarded as a set of three groups, namely, juveniles comprising immature individuals and two adult groups comprising mature females and males involved in the reproduction process. When developing a mathematical model, the following mechanisms are taken into account: 1) the dependence of newborn number on the ratio of the number of males to that of females in the population, and 2) density-dependent regulation of juvenile survival, when immature individual survival decreases with size growth of sex-age classes. The model considered may be written as a system of three equations:

$$\begin{cases} p_{n+1} = a\min(f_n, 2f_n m_n/(f_n/h + m_n)), \\ f_{n+1} = \delta(1 - \alpha_1 p_n - \beta_1 f_n - \gamma_1 m_n)p_n + sf_n, \\ m_{n+1} = (1-\delta)(1 - \alpha_2 p_n - \beta_2 f_n - \gamma_2 m_n)p_n + vm_n \end{cases}, \qquad (14)$$

where n denotes the ordinal number of the reproduction season; p signifies the number of immature individuals; f and m stand for the numbers of mature females and males, respectively; a is the birth rate; h is the average size of the harem; δ is the proportion of females among newborns; s and v are the survival rates of mature females and males, respectively; α_i, β_i and γ_i are coefficients characterizing the intensity of the competitive impact on the survival rate of immature and mature females and males, respectively.

We made the analytical and numerical research of the mathematical model. For making the numerical experiments, we have elaborated the software systems to construct of bifurcation diagrams, attraction basins, dynamic modes maps, map of eigenvalues, and Lyapunov exponents.

It is shown, with changing parameters' values and transition through the stability domain boundary the stability loss of the model fixed point may occur according to both scenarios: the Neimark-Sacker bifurcation scenario and the Feigenbaum scenario. Depending on the values of the model parameters, the transition to chaos can be realized through the destruction of the invariant curve or through the period doubling cascade. We consider evolution of strange attractors in the model. It is found that there are chaotic dynamics and hyperchaos in proposed model.

This work is in part supported by the Russian Foundation for Basic Researches (project no. 16-31-00218 mol_a) and the Fundamental Research Complex Program of FEB RAS.

Solution to the Cauchy problem for parabolic PDE using space translation based formulas

I. D. Remizov

Lobachevsky Nizhny Novgorod State University

ivremizov@yandex.ru

Consider $x \in \mathbb{R}^1$, $t \geq 0$ and set the Cauchy problem for a second-order parabolic partial differential equation

$$\begin{cases} u'_t(t,x) = (a(x))^2 u''_{xx}(t,x) + b(x)u'_x(t,x) + c(x)u(t,x) = Hu(t,x), \\ u(0,x) = u_0(x). \end{cases} \tag{1}$$

The coefficients a, b, c, u_0 above are bounded, uniformly continuous functions $\mathbb{R}^1 \to \mathbb{R}^1$. Assuming the existence of C_0-semigroup $\left(e^{tH}\right)_{t\geq 0}$ we obtain [1] the following formula for the solution:

$$u(t,x) = \left(e^{tH}u_0\right)(x) = \lim_{n\to\infty}\left(\Big(S(t/n)\Big)^n u_0\right)(x), \tag{2}$$

where

$$(S(t)f)(x) = \tfrac{1}{4}f\left(x + 2a(x)\sqrt{t}\right) + \tfrac{1}{4}f\left(x - 2a(x)\sqrt{t}\right) + \tfrac{1}{2}f(x + 2b(x)t) + tc(x)f(x). \tag{3}$$

Formula (3) can be rewritten in terms of generalized functions (=distributions) basing on the fact that $f(w) = \int_{\mathbb{R}} \delta(y-w)f(y)dy$:

$$(S(t)f)(x) = \int_{\mathbb{R}} \Big[\tfrac{1}{4}\delta\left(y - x - 2a(x)\sqrt{t}\right) + \tfrac{1}{4}\delta\left(y - x + 2a(x)\sqrt{t}\right) +$$

$$+\tfrac{1}{2}\delta(y - x - 2b(x)t) + tc(x)\delta(y-x)\Big] f(y)dy.$$

Employing this equality one can rewrite (2) as a Feynman formula in which the integral kernel is a distribution (=generalized function).

It also seems interesting to derive such formulas for the Schrödinger equation, this is not done, but similar results involving Quasi-Feynman formulas are already obtained, see [2-4].

[1] *I. D. Remizov.* Approximations to the solution of Cauchy problem for a linear evolution equation via the space shift operator (second-order equation example), arXiv:1605.03908

[2] *I. D. Remizov.* Quasi-Feynman formulas – a method of obtaining the evolution operator for the Schrödinger equation, Journal of Functional Analysis 270:12 (2016), pp. 4540-4557

[3] *I. D. Remizov* Solution of the Schrödinger Equation with the Use of the Translation Operator, Math. Notes, 100:3 (2016), 499-503

[4] *I. D. Remizov* New Method for Constructing Chernoff Functions, Differential Equations, 53:4 (2017), 566-570

On Takens Last Problem: times averages for heteroclinic attractors

A. Rodrigues

rodrigues@fc.up.pt

In this talk, after introducing some technical preliminaries about the topic, I will discuss some properties of a persistent family of smooth ordinary differential equations exhibiting tangencies for a dense subset of parameters. We use this to find dense subsets of parameter values such that the set of solutions with historic behaviour contains an open set. This provides an affirmative answer to Taken's Last Problem (F. Takens (2008) Nonlinearity, 21(3) T33–T36). A limited solution with historic behaviour is one for which the time averages do not converge as time goes to infinity. Takens' problem asks for dynamical systems where historic behaviour occurs persistently for initial conditions in a set with positive Lebesgue measure. The family appears in the unfolding of a degenerate differential equation whose flow has an asymptotically stable heteroclinic cycle involving two-dimensional connections of non-trivial periodic solutions.

We show that the degenerate problem also has historic behaviour, since for an open set of initial conditions starting near the cycle, the time averages approach the boundary of a polygon whose vertices depend on the centres of gravity of the periodic solutions and their Floquet multipliers. This is a joint work with I. Labouriau (University of Porto).

Dynamics of a network consisting of two rings of Henon maps and Lozi maps with nonlocally coupling

Rybalova E.V., Anischenko W.S.

Saratov National Research State University

rybalovaev@gmail.com, wadim@info.sgu.ru

In the present work we consider a system (network) consisting of two rings with the i-i coupling. The first ring represents nonlocally coupled two-dimensional Henon maps which exhibit a nonhyperbolic chaotic attractor. In this ensemble the transition from complete synchronization to complete desynchronization occurs through so-called chimera states. The second ring consists of nonlocally coupled two-dimensional Lozi maps which are characterized by a nearly hyperbolic chaotic attractor. In this case the synchronization-desynchronization transition takes place through solitary states. The control parameter values of the elements of the considered rings correspond to the chimera state regime (the Henon map ensemble) and solitary states (the Lozi map ensemble).

Our numerical studies have shown that in the case of two coupled rings with different elements, the ring of Henon maps can demonstrate solitary states while phase chimera states can be observed in the ring of Lozi maps. If the parameter α value in the Henon map corresponds to the periodic motion of the ring, the model under consideration can exhibit regimes which have not been observed earlier in the individual (separate) rings.

Finally, in the present work we also consider the influence of the dynamics of the Lozi map ring on the lifetime of the amplitude chimera in the ring of Henon maps. Our numerical simulation has shown that in this case, the amplitude chimera can "live" infinitely long.

Nonconservative reversible perturbations of reversible maps with unit Jacobian

Evgeniya Samylina

Lobachevsky State University of Nizhny Novgorod

samylina_evgeniya@mail.ru

In this work, using the method proposed in [1], we construct and investigate a reversible non-conservative map which is a perturbation of square of cubic conservative reversible Hénon map investigated in [2]. Also in [2] it was shown that resonance 1:4 points can undergo a pitch-fork bifurcation, due to which a symmetrical (lying on the line of fixed points of involution) point is divided into 2 elliptic points and a symmetrical saddle point. Here we show that in the case of the non-conservative reversible map such bifurcation leads to appearance of an asymptotically stable, completely unstable and saddle point from the symmetrical elliptic point. Thus, we illustrate theorem from paper [3] which approves that resonance zones near an elliptic periodic point of a reversible map must, generically, contain asymptotically stable and asymptotically unstable periodic orbits, along with wild hyperbolic sets.

[1] Delshams A. et al. Abundance of attracting, repelling and elliptic periodic orbits in two-dimensional reversible maps //Nonlinearity. 2012. Vol. 26. No.1. p. 1.

[2] Gonchenko M., Gonchenko S., Ovsyannikov I. Bifurcations of cubic homoclinic tangencies in two-dimensional symplectic maps //Mathematical Modelling of Natural Phenomena. 2017. Vol. 12. No. 1. p. 41-61.

[3] Gonchenko S. et al. Attractors and repellers near generic elliptic points of reversible maps //arXiv preprint arXiv:1212.1931. 2012.

Diffusion in Hilbert space endowing with the translationary and rotationary invariant measures

V. Sakbaev

Moscow Institute of Physics and Technology

fumi2003@mail.ru

Hamiltonian second quantization

Shamarov Nikolaj

nshamarov@yandex.ru

Saga of modeling biological central pattern generators

A. Shilnikov

Georgia State University, USA; Lobachevsky State University of Nizhny Novgorod

ashilnikov@gsu.edu

Transformations of the Lebesgue and Feynman generalized measures and quantum anomalies

O.G.Smolyanov

Moscow University, Russia

smolyanov@yandex.ru

The differential calculus for (generalized) measures on infinite dimensional functional spaces will be applied to discuss the contradiction between the points of view on quantum anomalies presented in the books by Fujikawa and Suzuki (K.Fujikawa and H.Suzuki, Path Integrals and Quantum Anomalies. Oxford, 2004, 2013) on the one hand, and by Cartier and DeWitt-Morette (P.Cartier and C.DeWitt-Morette, Functional Integration: Action and Symmetries. CUP, 2006) on the other.

Dynamics of QR flow

J.-C. Tatjer

Barcelona University, Spain

jcarles@maia.ub.es

Excitation and stability of waves in discrete complex Ginzburg-Landau equation: effects of spatial inhomogeneity

Tikhomirov A.A.

Lobachevsky State University of Nizhny Novgorod

andreynn2006@yandex.ru

Recently phenomenon of exciton-polariton condensation in quantum wells formed by semiconductor microcavities bordered by Bragg mirrors has been found. These systems are of fundamental interest as a new realization of Bose-Einstein condensates. Classical description of these systems is possible in the form of discrete Ginzburg-Landau equation (DGLE). In this research some features of dynamics in DGLE system with diatomic inhomogeneity are studied by means of mode formalism. In particular, a set of two-mode invariant manifolds is found. Modulational instability of plane waves is investigated. The most stable wave has the wave number $k_0 = 0$ in the binary node basis. It is shown that in comparison with spatial homogenous system diatomic inhomogeneity demands decreased dissipation or increased pumping for plane waves existence. Besides, at high conservative nonlinearity such inhomogeneity may lead to stabilization of waves which would be unstable in the homogeneous case. When conservative nonlinearity is gradually increased, several crossovers between different dynamical regimes are observed as follows: harmonic wave, then quasiperiodical regime, localized chaos and global chaos in mode space.

On non-trivial wandering domains

D. Turaev

Imperial College, London

d.turaev@imperial.ac.uk

Chimeras in ensembles of bistable oscillators

T.E. Vadivasova, I.A. Shepelev

Department of Physics, Saratov State University,

Astrakhanskaya str., 83, 410012, Saratov, Russia

vadivasovate@yandex.ru

We consider ensembles of bistable elements with nonlocal interaction. It is shown that bistability in the case of nonlocal interaction leads to formation of chimera structures of a special type, which we have called double-well chimeras. Their distinctive feature consists in the formation of incoherent clusters with irregular distribution of elements between the two attractive sets existing in individual element (two "potential wells"). A type of bistability of an individual element can be different. In the simplest case, when a bistable element has two stable equilibrium points, it is possible to observe the fixed double-well chimera-like structures for a certain values of coupling parameters. In this case, the elements of ensemble do not oscillate in time. For the ensemble of bistable FitzHugh-Nagumo oscillators the double-well chimeras can be obtained that are not completely immobile. In this case, the oscillators belonging to incoherent clusters oscillate in time. These oscillations can be either periodic or chaotic.

The ensemble of cubic maps was also considered in the regime of chaotic dynamics. In this case, a variety of spatio-temporal regimes is observed for changing of the coupling parameters, including chimera structures of different types. In addition to fixed double-well chimeras, it is possible to observe the double-well chimeras, for which periodic or weakly chaotic behavior of the elements in time takes place. For certain values of coupling, the single-well chimera structures exist. They are similar to chimeras, found in ensembles of oscillators and return maps with the Feigenbaum scenario of chaos development. Besides the ensemble of cubic maps, the ensemble of Chua oscillators was considered. The qualitative conformity of the results for these models is shown.

The work was financially supported by the Ministry of Education and Science of the Russian Federation within the framework of the state task (project code 3.8616.2017 / BC).

On inverse Klein-Gordon problem for dublet of dions

Gavrilin A. T.
National Research Lobachevsky State University of Nizhny Novgorod; <gavrilin@rf.unn.ru>

В предыдущих работах автора [1,2], была предложена модель заряженных лептонов (электрона, мюона и тауона) в форме двух сильно взаимодействующих друг с другом магнитных зарядов противоположного знака - монополей Дирака. В рамках этой модели магнитный момент лептонов имел чисто статическую природу и не связывался с вращением электрического субстрата, а механический момент (спин) вообще отрицался. В свое время Швингер ввел понятие диона – гипотетической частицы, несущей как электрический, так и магнитный заряд. Здесь в духе Швингера каждый из монополей Дирака наделяется еще и электрическим зарядом, равным половине заряда электрона.

Динамика подобной системы исследовалась в [3] в рамках классической нелинейной динамики и физически правдоподобного потенциала взаимодействия дионов. Однако решения возникающего уравнения типа Дуффинга и дающие правильный спектр масс лептонов, обладали тем недостатком, что изменения расстояния между дионами на некоторых временных интервалах происходили со сверхсветовой скоростью.

В связи с изложенными обстоятельствами, по-видимому, целесообразно изучить предложенную модель в рамках релятивистской квантовой механики. Требуется по известным из экспериментов спектру масс лептонов и временам жизни мюона и тауона получить потенциал сильного взаимодействия между магнитными зарядами. Известно, что сколько-нибудь содержательные обратные задачи относятся к классу некорректно поставленных и, в частности, допускают целое множество возможных решений. Регуляризация (уменьшение неопределенности решения) подобных задач, так или иначе, сводится к использованию априорной информации о классе возможных решений.

Постулируется, что дионы являются скалярными частицами и их сильное магнитное притяжение не является кулоновским, а характеризуется потенциалом, отличающимся от потенциала Юкавы множителем в виде линейной комбинации четырех полиномов Эрмита низшего порядка. Поскольку потенциал взаимодействия дионов цилиндрически симметричен, уравнение Клейна-Гордона (УКГ) рассматривается в 2D пространстве. В цилиндрическом базисе $\left\{\vec{e}_\rho, \vec{e}_\varphi, \vec{k}\right\}$ «составной» 4-потенциал взаимодействия имеет вид $\left(U(z)+\dfrac{e^2}{4z},\ \dfrac{\mu_0}{4\pi}\vec{e}_\varphi\right)$, где μ_0 - магнетон Бора, а стационарное уравнение УКГ -

$$\left[-\hbar^2 c^2\left(\frac{\partial}{\partial\rho^2}+\frac{1}{\rho}\frac{\partial}{\partial\rho}\right)+c^2\left(-i\frac{\hbar}{\rho}\frac{\partial}{\partial\varphi}-\frac{e\mu_0}{4\pi\rho^2}+\frac{e^2}{4z}+U(z)\right)\right]\psi = E^2\,\psi\,,$$

где $U(z)$ - подлежащий определению потенциал сильного взаимодействия дионов.

Результаты «условного варьирования» соответствующего лагранжиана приводят к потенциалу $U(z)$, для которого, к сожалению, УКГ дает метастабильные состояния (мюон, тауон), времена жизни которых далеки от экспериментальных.

[1]. Гаврилин А.Т. // В кн. Труды XX-й научн. конф. по радиофизике 16 мая 2016г. Н. Новгород, 2016. с.261.
[2]. Gavrilin A.T. // Conf.-School “Dynamics, Bifurcation and Chaos. 2016”. N. Novgorod, Russia, July 18-22, 2016. Book of Abstracts. P.11.
[3] Gavrilin A.T. // Shilnikov WorkShop 2016. 16-17 December of 2016 Book of Abstracts. P.18.