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Л.П. ШИЛЬНИКОВА
(1934-2011)

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BOOK of ABSTRACTS

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On some features of coupled discrete Rössler oscillators dynamics

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In this work we consider the dynamics of two coupled systems with quasi-periodic oscillations. The investigation of such systems became rather popular last years.

We use the system of two Rössler oscillators [1] with linear coupling and apply the discretization procedure [2] to obtain the system of coupled discrete Rössler oscillators:

$$
\begin{aligned}
x_{n+1} &= x_n - \varepsilon(y_n + z_n),\\
y_{n+1} &= y_n + \varepsilon(x_n + a_1 y_n) + \varepsilon\mu(v_n - y_n),\\
z_{n+1} &= z_n + \varepsilon b + \varepsilon(x_n - r)z_n,\\
u_{n+1} &= u_n - \varepsilon(v_n + w_n),\\
v_{n+1} &= v_n + \varepsilon(u_n + a_2 v_n) + \varepsilon\mu(y_n - v_n),\\
w_{n+1} &= w_n + \varepsilon b + \varepsilon(u_n - r)w_n.
\end{aligned}
$$

The main advantage of suggested system is that the subsystems demonstrate the quasi-periodic oscillations in the wide range of the parameters and the synchronization tongues are extremely narrow so it seems to be convenient to use it for investigation of phenomena specific to coupled quasi-periodic oscillators.

We revealed that the regions of quasi-periodic oscillations with 2 frequencies (2D tori) form the resonance web (for small coupling) or the system of tongues (for larger coupling) embedded into the regions of quasi-periodic oscillations with 3 frequencies (3D tori) in the parameter plane (a_1, a_2). Also we observe doublings of 2D and 3D tori and the attractors with complex structure and close to zero largest Lyapunov exponent.

The work was supported by RFBR (projects No. 12-02-31089 and 12-02-00541).

References

1.Rössler O.E. // Phys. Lett. 1976. V. A57, № 5. P. 397.

2.G.M. Zaslavsky. Physics of Chaos in Hamiltonian Systems. Imperial College Press, 1998.

Directional Complexity and Entropy for Lift Mappings

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We introduce and study the notion of a directional complexity and entropy for maps of degree 1 on the circle. For piece-wise affine Markov maps we use symbolic dynamics to relate such a complexity to the symbolic complexity. We apply a combinatorial machinery to obtain exact formulas for the directional entropy, and to find the maximal directional entropy, and to show that it equals to the topological entropy of the map.

Discrete spectrum of nonlinear modes: a mechanism to emerge

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We discuss a hypothesis on the existence of a countable set of heteroclinic orbits connecting saddle-center points (also called "embedded solitons" in some applications). In short, it can be described as follows.

Let a system of differential equations depend on some external parameter ε that defines a singular perturbation. Assume that the system has a heteroclinic orbit for $\varepsilon = 0$ and that the corresponding solution can be analytically extended into upper complex half-plane with the closest to the real axis singularities given by a pair of points $z = \pm\alpha + i\beta$. Then there is a countable set of heteroclinic orbits for the singularly perturbed system corresponding to the discrete set of values ε, $\varepsilon = \varepsilon_1, \varepsilon_2, \varepsilon_3, \ldots$ such that $\varepsilon_n \to 0$ as $n \to \infty$ and

$$\varepsilon_n \sim \frac{\beta}{\pi/2 + n\pi + \varphi_0},$$

where φ_0 is a constant. We illustrate this statement by numerical results for several nonlinear problems of various physical origin.

This was done in collaboraion with E.V. Medvedeva and D.E. Pelinovsky.

Librations of Hamiltonian systems and tunnelling in quantum double well

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We study a classical natural system with the standard kinetic energy and a potential with two symmetric maxima. It is shown that under natural non-degeneracy hypotheses the above system has a one-parameter family of periodic solutions lying near a trajectory heteroclinic to the two maxima. The periodic solutions happen to be librations (i.e. they oscillate between the endpoints). We derive an asymptotic formula for the difference between Maupertuis actions on the doubly asymptotic trajectory and on a nearby libration. To this end, we essentially use a construction due to Shilnikov [4].

As an application, we obtain a formula for the exponentially small (as $h \to 0$) splitting of the lowest eigenvalues of a Schrödinger operator $-\frac{1}{2}h^2\Delta + V(x)$ with a potential having two symmetric wells. We give thus the mathematical justification to an analogous result [3], where the above splitting formula was derived at the physical level of rigor.

References

[1] A.Yu. Anikin, Asymptotic behaviour of the Maupertuis action on a libration and a tunneling in a double well. I. Rus. J. of Math. Phys. 2013. V. 20. No. 1. (to appear)

[2] A.Yu. Anikin, Libration and splitting of the ground state in multidimensional double well problem. Theoret. and Math. Phys. 2013. V. 175. No. 2. (to appear)

[3] J. Brüning, S. Yu. Dobrokhotov, E. S. Semenov. Unstable closed trajectories, librations and splitting of the lowest eigenvalues in quantum double well problem. Regul. Chaotic Dyn. 2006. V. 11. No. 2. P. 167–180.

[4] L.P. Shilnikov, On a Poincaré–Birkhoff problem. Math. USSR Sbornik. 1967. V. 3. P. 353–371. (in Russian)

Poincaré recurrences and their applications in nonlinear dynamics

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The basic statistical characteristics of Poincaré recurrence sequence are obtained numerically for the logistic map in a chaotic regime. The mean values, variation and recurrence distribution density are calculated and their dependence on a return size is analyzed. Afraimovich-Pesin dimension values are obtained. It is verified that the Afraimovich-Pesin dimension corresponds to the Lyapunov exponent. The peculiarities of the influence of noise on the recurrence statistics are studied in local and global approaches. It is shown that the obtained numerical data fully conform to the theoretical results. It is demonstrated that the Poincaré recurrence theory can be applied to diagnose effects of stochastic resonance and chaos synchronization and to calculate the fractal dimension.

On computation of Kolmogorov-Sinai entropy

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Let $(\Omega, \mathcal{F}, \mu)$ be a probability space and $T : \Omega \to \Omega$ be a measure preserving tranformation. If $\mathcal{A}$ and $\mathcal{B}$ are finite partitions of Ω then we define partitions

$$\mathcal{A} \vee \mathcal{B} = \{A \cap B \mid A \in \mathcal{A},\ B \in \mathcal{B}\}, \qquad T^{-1}\mathcal{A} = \{T^{-1}(A) \mid A \in \mathcal{A}\}.$$

Let also $H_\mu(\mathcal{A}) = -\sum\limits_{A \in \mathcal{A}} \mu(A) \log \mu(A)$ be the entropy of $\mathcal{A}$. Then the *Kolmogorov-Sinai entropy* of a map T is defined by

$$h^{KS}(T) := \sup_{\mathcal{A}} \lim_{k \to \infty} \frac{1}{k} H_\mu \left(\bigvee_{i=0}^{k-1} T^{-i}\mathcal{A} \right),$$

where $\mathcal{A}$ runs over *all* finite partitions of Ω.

Let $\xi : \Omega \to R^n$ be a measurable map. Using it we can define a sequence of so-called *ordinal partitions* $\{\mathcal{P}_d^{\xi,T}\}_{d\geq 1}$ of Ω. In the paper [1] it was shown that

$$h^{KS}(T) = \lim_{d\to\infty} \lim_{k\to\infty} \frac{1}{k} H_\mu \left(\mathcal{P}_d^{\xi,T}\right) \tag{1}$$

under assumptions that Ω is a smooth manifold of dimension m, T is a μ-preserving *diffeomorphism*, $n > 2m$, and $\xi : \Omega \to R^n$ is an *embedding.*

Theorem. *Let Ω be a smooth manifold of dimension m, μ be a measure on the Borel σ-algebra $B(\Omega)$, $T : \Omega \to \Omega$ be a measurable μ-preserving transformation. Suppose also that μ is Lebesgue absolute continuous. Then the set of all $\xi \in C^\infty(\Omega, R^n)$ for which (1) holds is residual in $C^\infty(\Omega, R^n)$ provided $n > m$.*

References

[1] K. Keller, *Permutations and the Kolmogorov-Sinai entropy*, Discrete Contin. Dyn. Syst. **32** (2012), no. 3, 891–900.

[2] A. Antoniouk, K. Keller, S. Maksymenko, *Kolmogorov-Sinai entropy via separation properties of order-generated σ-algebras*, `arXiv:1304.4450`

Shilnikov bifurcations in the Hopf-zero singularity

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The Hopf-zero singularity consists in a vector field in $\mathbf{R}^3$ having the origin as a critical point, with a pure imaginary pair and a simple zero eigenvalue. For some of its unfoldings, one can see [1] that the truncation of the normal form at any order has two saddle-focus critical points with a one- and a two-dimensional heteroclinic connection.

However, if one considers the whole vector field, one expects these heteroclinic connections to be destroyed. This can lead to the birth of a homoclinic connection to one of the critical points, producing thus a Shilnikov bifurcation. For the case of $\mathcal{C}^\infty$ unfoldings, this has been proved before [2], but the analytic case is still an open problem.

Recently, it has been seen in [3] that a complete understanding of the splittings of the heteroclinic connections is the last step to prove the existence of Shilnikov bifurcations for the analytic case. Our work concerns the study of these splittings (see [4] for the one-dimensional case).

References

[1] Guckenheimer, J. and Holmes, P., *Nonlinear oscillations, dynamical systems, and bifurcations of vector fields.* Springer-Verlag, New York (1983), 376–396.

[2] Broer, H. W. and Vegter, G., Subordinate Šil'nikov bifurcations near some singularities of vector fields having low codimension. *Ergodic Theory Dynam. Systems*, **4** (1984), 509–525.

[3] Dumortier, F., Ibáñez, S., Kokubu, H. and Simó, C., About the unfolding of a Hopf-zero singularity. *Preprint.*

[4] Baldomá, I., Castejón, O. and Seara, T. M., Exponentially small heteroclinic breakdown in the generic Hopf-zero singularity. *To appear in the "Journal of Dynamics and Differential Equations"*. doi: 10.1007/s10884-013-9297-2

Bifurcation of double symmetric periodic solutions of a Hamiltonian system

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We consider a Hamiltonian system with two degrees of freedom which is invariant under finite group $\mathcal{G}$ with two generators g_1 and g_2 of linear transformations of extended phase space. Such situation is rather common for many problems of celestial mecha-nics and cosmodynamics. We investigate the dynamics near doubly symmetric periodic solution and provide its bifurcation analysis as well.

Dynamics near a periodic solution is described in the linear approximation by monodromy matrix M which is the solution of Cauchy problem for variational equation along the original periodic trajectory. M has additional internal symmetries in the case of symmetry of corresponding solution. The original solution, called after Poincaré as the solution of the first kind, is orbitally stable if its stability index $S \equiv (\mathrm{T}r\, M - 2)/2$ satisfies inequality $|S| < 1$. We prove that stability index of double symmetric periodic solution with period T is not less then -1. If $S = \cos(2\pi p/q)$, where p, q are integer, there exist periodic solutions with period qT, called the solution of the second kind. We state that: 1) if both p and q are odd, there exists one double symmetric periodic solution of second kind with period equal to qT; 2) if at least one of the numbers p or q is even, there exist two pairs of single symmetric solutions of second kind, which are mutually symmetric to each other.

Two different scenarios of period doubling bifurcation (i. e. $S = -1$) of double symmetric periodic solution may occur. The first one is when two pairs of symmetric periodic solutions of the second kind with one symmetry both finish at double symmetric solution of the first kind. The second one is when one pair of periodic solutions of second kind with one symmetry finishes at double symmetric periodic solution and other pair of periodic solutions of second kind with other symmetry starts from it.

We apply the above results to study the families of periodic solutions of Hill's problem.

Method of confidential domains in the analysis of noise-induced transitions for Goodwin model

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The Goodwin dynamical model [1] under the random external disturbances is considered

$$\ddot{x}(t) + a\frac{x(t)^2 - 1}{x(t)^2 + 1}x(t) - bx(t) + cx^3(t) = \varepsilon\dot{w}(t)$$

where the variable x is a deviation of national income from the equilibrium, parameters a, b, c are positive, w is a standard Wiener process, ε is a additive noise intensity.

A full parametrical analysis for equilibria and cycles of deterministic model is developed. The phenomenon of the birth the stable cycle in parametrical zone where equilibria are stable is investigated. The analysis of the separatrix dividing basins of attraction is carried out numerically. We study probabilistic properties of stochastic attractors using method of confidential areas based on the stochastic sensitivity functions technique [2]and numerical methods. A phenomenon of the generation of stochastic business cycles in the zones of stable equilibria and noise-induced transitions between stable attractors is discussed.

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References

[1] *R.M.Goodwin* The Nonlinear Accelerator and the Persistence of Business Cycles. // Econometrica, Vol 19, No 1 (Jan., 1951), Pp 1-17.
[2] *Bashkirtseva I., Ryashko L.* Stochastic sensitivity analysis of noise-induced excitement in a prey-predator plankton system. // Frontiers in Life Science, 2011, Vol 5, Pp 141-148.

On dynamics of the quadratic trace map

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We consider the quadratic map

$$F(x,\, y) = (xy,\, (x-2)^2), \tag{1}$$

where $(x,\, y)$ is a point of the plane $\mathbf{R^2}$.
Denote by Δ the closed triangle $\{(x;y) : x \in [0,4], y \in [0,4], x+y \leq 4\}$.

The following theorem is proved here.

Theorem. *Let F_μ be a map of the type (1). Then the nonwandering set $\Omega(F)$ coincides with the triangle Δ that is the mixing repeller with everywhere dense periodic point set.*

This is the joint work with L. S. Efremova.

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References

1. S. Bel'mesova, L. Efremova. On unbounded trajectories of a certain quadratic mapping of the plane// J. Math. Sci. (N.Y.). – 2009. – V. 157 (3). – P. 433 - 441.

2. Belmesova S. S., Efremova L. S. On invariant sets of some quadratic mapping of the plane// Vestnik University of Nizhni Novgorod. – 2012. – №2 (2). – P. 152 - 158 (In Russian).

Ghost attractors in randomly switched dynamical systems

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In this work, we develop a general rigorous theory of stochastically switched dynamical systems and apply rigorous mathematical techniques to investigate the interplay between overall system dynamics and the stochastic switching process. We study dynamical systems and networks whose coupling or internal parameters switch on and off randomly, and the switching time is fast, with respect to the characteristic time of the individual node dynamics. If the stochastic switching is fast enough, we expect the switching system to follow the averaged system where the dynamical law is given by the expectation of the stochastic variables. We ask the question under what conditions a solution of the randomly switched system converges to an attractor of the averaged system. The answer contains various subtleties and depends on whether or not the attractor in the averaged system is unique and whether it is an invariant set for all switching sequences. In the non-invariant single attractor case, the trajectories of the switching system reach a neighborhood of the attractor rapidly and remain close most of the time with high probability when switching is fast. In this case, the attractor of the averaged system acts as a ghost attractor for the switching system. In the non-invariant multiple attractor case, the trajectory may escape to another ghost attractor with small probability. Using the Lyapunov function method, we derive explicit bounds that relate these probabilities, the switching time, and dynamical systems' parameters.

Multidimensional Lurie systems and Henon maps: Smale's horseshoes and bifurcations

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In this talk we consider the Lurie system (system with one scalar nonlinearity) $Dy = Ay + b\varphi(x), x = C^T y$, where unimodal function $\varphi : R^1 \to R^1$, $A - n \times n$ matrix and $D = \dfrac{d}{dt}$ or $Dy(t) = y(t+1)$ for continuous and discrete time respectively.

In the case of continuous time, using method of 2-d comparison systems we prove the existence of homoclinic orbit of the saddle-focus, which due to the Shilnikov's theorem provides the chaotic dynamics.

In the case of discrete time we obtain the normal form of the Lurie system as the following map $F : \begin{pmatrix} \overline{x} \\ \overline{y} \end{pmatrix} = \begin{pmatrix} 1 & \mathbf{1} \\ 0 & B \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} a \\ b \end{pmatrix} \varphi(x)$, where $B-(n-1)\times(n-1)$-matrix and $\mathbf{1} = (1, 1, ..., 1)$. For the unimodal continuous functions $\varphi(x)$ having a bounded away from zero discontinuous derivative we prove the existence of a parameter domain d_{hyp} for which the map F has a singularly hyperbolic attractor. In the case of unimodal smooth functions we consider the limiting sets of F, which can be studied in terms of symbolic dynamics.

Theorem. *There exists a parameter domain d_{sh} such that the map $F|_{d_{sh}}$ has a multidimensional Smale's horseshoe.*

The last example we consider is the generalized Henon map $\overline{x} = f(x) + \sum_{j=1}^{n} a_j v_j$, $\overline{v}_j = v_{j-1}$, $v_0 \equiv x$. The variables change $v_j = a_j^{-1} y_j + x$ transforms this map to the map F, for which above theorem is valid under the conditions: $|a_1|$ is small and $|\dfrac{a_j}{a_{j-1}}| < q < 1$. We find a parameter domain d_{ca} for which both the map F and Henon map have a chaotic attractor. The bifurcations corresponding to transition $d_{ca} \to d_{sh}$ is studied.

Finally, we discuss the case of the other types of nonlinearities.

Statistical Characteristics of Poincare Return Times within the Local Approach under External Force and Noise Conditions

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Our purpose is to analyze the statistical characteristics of a sequence of Poincaré return times in an ϵ-neighborhood of a chosen initial state x_0 for a chaotic attractor of a dynamical system (in the framework of the local point-wise approach).
Experimental data are taken by iteration of a one-dimensional map – logistic map:

$$x_{n+1} = rx_n(1 - x_n) \tag{1}$$

The dependence of the mean time for Poincaré return times, the distribution density of the attractor of system and the distribution density of the Poincaré first return times to the ϵ-neighborhood of the initial state x_0 have been analyzed both in the absence and in the presence of external noise (2) and external force (3)

$$x_{n+1} = rx_n(1 - x_n) + \sqrt{2D}\xi(t) \tag{2}$$

where D is the intensity of external white noise $\xi(t)$

$$x_{n+1} = rx_n(1 - x_n) + A \sin \Omega n. \tag{3}$$

Shilnikov lemma for a nondegenerate critical manifold of a Hamiltonian system

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We consider a Hamiltonian system with a nondegenerate normally hyperbolic symplectic critical manifold $M \subset H^{-1}(0)$ and prove an analog of Shilnikov lemma (or strong λ-lemma). We use it to show that for small μ, certain chains of heteroclinic orbits to M can be shadowed by a trajectory on $H^{-1}(\mu)$. This is a generalization of a theorem of Shilnikov and Turaev. An applications to the Poincaré second species solutions of the 3 body problem will be given.

The hierarchy of the dynamics of a body rolling without slipping and spinning on a plane

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The paper is concerned with a comprehensive analysis of the dynamics of various rigid bodies rolling without slipping and spinning on a plane. It is shown that a hierarchy of types of dynamical behavior arises depending on the existence of tensor invariants (in this case – the first integral, the invariant measure and the symmetry fields). Thus, depending on the surface of the rolling body and its mass distribution some systems exhibit regular behavior typical of completely integrable conformally Hamiltonian systems, whereas the most asymmetric bodies exhibit a chaotic behavior typical for dissipative systems (in particular, the appearance of strange attractors is possible). Several new integrable systems of nonholonomic mechanics are pointed out. This paper is of particular importance for the control and design of mobile robots rolling on the surface since one can point out systems exhibiting both the most simple and the most complicated behavior as well as choose the most optimal form of the surface of rolling for the problems of interest.

The model of economic dynamics with discrete time

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We consider the model with overlapping generations and altruism that realized as 3-dimensional dynamical system with discrete time. O.Galor [1] investigated hypothesis about a perfectly competitive world where economic activity is performed over infinite discrete time. In every period, two goods, a physical (consumer) good and an investment good, are produced using two factors, labor and capital, in the production process, and live two generations: working individuals and retirees. This situation defines two-sector overlapping-generations model with two generations (the book [2] include detailed exposition about overlapping-generations model).

We develop two-sector overlapping-generations model in this paper in situation when there are three generations in every period: children, working individuals and retirees. We describe the structure of incomes and financial flows between generations (the example of overlapping-generations model with altruism see in [3]). We suppose that the life-cycle utility function is assumed to be additively separable and consider constant inter-temporal elasticity of substitution instantaneous utility function. We suppose that the production function is homogeneous of degree one and consider Cobb-Douglas production function. We consider dynamics of capital and price of consumer good, and get the system of difference equations of third order in given conditions.

References

[1] Galor O. A Two-Sector Overlapping-Generations Model: A Global Characterization of the Dynamical System // Econometrica, Vol. 60, No. 6. (Nov., 1992), pp. 1351-1386.
[2] De la Croix D., Michel P. A Theory of Economic Growth Dynamics and Policy in Overlapping Generations. Cambridge University Press, 2004.
[3] Michel P. and Pastieau P. Fiscal Policy with Agents Differing in Altruism and Ability // Economica, New Series. 2005. Vol. 72. №285. P.121-135.

On the conservation of the magnetic moment in fusion plasma

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In tokamak devices, one often assumes that the adiabatic invariant is actually a real constant of the motion. This allows as a first approximation to link particles trajectories to magnetic field lines. In this talk, we will discuss the motion of charged particles in a magnetic field and discuss the chaotic nature of trajectories in two situations. In the first case we shall consider a magnetic configuration with a cylindrical symmetry. While in the second case we shall consider a perturbation and a ripple effect which is non generic as magnetic filed lines are still integrable. But, this perturbation destroys one constant of the motion, opening questions between the link of chaotic magnetic lines and chaotic particle motion.

Homoclinic snaking; localised pattern formation in Hamiltonian and reversible systems

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This talk shall consider theory and applications of a codimension-one mechanism for the generation of infinitely many homoclinic orbits in reversible dynamical system such as frequently arise as the equilibrium equations for pattern formation PDEs on infinite spatial domains. The mechanism involves the unfolding of a pair of heteroclinic tangencies between a symmetric hyperbolic equilibrium and a symmetric hyperbolic periodic orbit. This gives rise to a pair of curves that snake back and forth in a one-parameter bifurcation diagram on which homoclinic orbits occur that have arbitrarily many loops close to the periodic orbit. The bifurcation diagram is analogous to that for a curve of periodic orbits in the so-called Shilnikov-Hopf scenario. I shall review the state of the art of this theory and also point to analogues for PDEs in higher-dimensional space for which dynamical systems arguments do not readily apply. I shall finally turn to two recent extensions to the theory. The first, in joint work with Edgar Knobloch, Yi-Ping Ma and the late Thomas Wagenknecht concerns the situation where the turning points of the snake are caused not by a heteroclinic tangency, but by a saddle-centre point of the underlying periodic orbit. The second, in joint work with Victor Brena Medina concerns application of the homoclinic snaking idea to systems of reaction diffusion equations with a source term, arising in where the underlying Turing bifurcation is sub-critical. For all cases, I shall illustrate the theory with numerical examples applied to models that arise in applications.

Constructing a Simple 3D Autonomous Chaotic System with Arbitrary Numbers of Equilibria or Attractor-Scrolls

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In a typical 3D autonomous chaotic system, such as the Lorenz and Rossler systems, the number of equilibrium points is three or less and the number of scrolls in their attractors is two or less. Today, we are able to construct a simple 3D autonomous chaotic system that can have any desired number of equilibrium points or any desired number of scrolls in its attractor. This talk will briefly introduce the ideas and methodologies.

Optimal stationary cyclic exploitation of renewable resources

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Optimal exploitation of renewable resources is one of the key current problem, which is studied by various approaches [1], [2], [3], [4]. We optimize cyclic exploitation of renewable resources distributed on the circle. Our motion on the circle S^1 is described by a continuous control system with a set of control parameter U, which has at least two different points and is a compact manifold with boundary or disjoint union of them. An *admissible velocity* at a point $x \in S^1$ is a one provided by some value of control parameter. We assume that all admissible velocities are positive, and so due continuity of the family and compactness of S^1 and U there exists positive minimum v_0 of all admissible velocities on the circle.

An *admissible motion* is absolutely continuous map $t \mapsto x(t)$ from a time interval to the circle such that its derivative at any point of its differentiability belongs to the convex hull of all admissible velocities at the point $x(t)$. It is clear that any admissible motion could be extended to the all time axis, and due to $v_0 > 0$ any such extension provides cyclic rotation on the circle.

Passing by a point of circle we harvest a part of the resource located at this point. This part depends on our velocity and characteristics of the point. The natural optimization problem is to find an admissible motion which maximize time averaged profit on the infinite horizon.

After the harvesting the renewable resource recovers with some law. The talk will be devoted to existence of optimal stationary mode of exploitation, the stability of the respective resource dynamic and necessary optimality conditions. It will include results obtained recently in the team with T.Shutkina, A.Platov, A.Belyakov and V. Veliov.

The studies was supported by The Ministry of education and science of Russian Federation, projects DDPTS & T 1.1348.2011 and 14.B37.21.0362.

References

[1] W.Brock, A.Xepapadeas, Diffusion-induced instability and pattern formation in infinite horizon recursive optimal control// Journal of Economic Dynamics & Control 32 (2008), 2745 - 2787.

[2] A.A.Davydov, A.S.Platov, Optimal stationary solution in forest management model by accounting intra-species competition //Moscow Mathematical Journal, Vol. 12, No 2, April-June 2012, Pages 269-273.

[3] M.D.Smith, J.N.Sanchirico, J.E.Wilen, The economics of spatial-dynamic processes: Applications to renewable resources//J. Environmental Economics and Management 57 (2009), 104-121.

[4] A.A.Panesh, A.S.Platov, Optimization of size-structured population with interacting species// Journal of Mathematical Sciences, Vol. 188, No. 3, January, 2013.

Global instability in the elliptic restricted three body problem using two scattering maps

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The goal of the talk is to show the existence of global instability in the elliptic restricted three body problem. The main tool is to combine two different scattering maps associated to the normally parabolic infinity manifold to build trajectories whose angular momentum increases arbitrarily. The computation of such scattering maps will rely on previous computations for the circular case carried out by Llibre and Simó in 1980, which were extended to the elliptic case by Martínez and Pinyol in 1994.

This is a joint work with V. Kaloshin, A. de la Rosa and T.M. Seara.

Mixed dynamics in two-dimensional reversible maps

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Reversible systems, often positioned between conservative and dissipative, share important characteristics with both of them. This work is focused on two-dimensional reversible maps having heteroclinic cycles of symmetric saddle points intersecting non-transversally. For such a systems, one-parameter families of reversible maps unfolding that initial heteroclinic tangency are considered and it is proved the existence of infinitely many sequences (cascades) of bifurcations and the birth of asymptotically stable, unstable and elliptic periodic orbits. Roughly speaking, it is a mixed dynamics.

This result, joint to a previous work of Lamb and Stenkin, goes in the direction of proving a Reversible mixed dynamics conjecture, namely, that two-dimensional reversible maps with mixed dynamics are generic in Newhouse regions where maps with symmetric homoclinic and/or heteroclinic tangencies are dense.

Basin boundary bifurcations and hidden oscillations in multistable dynamics of controlled aero-elastic wing

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We consider mathematical model of an aircraft wing that is aeroelastically deformed by the airflow. A computer-based feedback control loop is introduced to suppress wing oscillations (called flutter) or to intentionally deform the wing to maximize air vehicle performance. The feedback law uses one or multiple flight control surfaces attached to the wing.

In open-loop four-dimensional model (without feedback) we have one global attractor, which can be the stable origin (i.e. zero point attractor) or a limit cycle in case of flutter. In controlled model under chosen feedback the qualitative picture is quite more complex. The stable origin can possess bounded basin of attraction and co-exist with stable limit cycle. Moreover, due to the bad choice of feedback law it can produce more than one stable point attractor or limit cycle with small amplitude near the origin. One can classify these limit cycles as "hidden oscillations".

In our dynamical system acting under feedback loop, it is advantageous to choose feedback parameters so as to eliminate multistability and make the origin its only global attractor. To achieve global suppression of wing oscillations, we study bifurcation sequence leading to limit cycle elimination, including basin boundary bifurcations. We attribute the global elimination of limit cycle to the boundary crisis, when its basin of attraction touches the limit cycle itself. To guarantee the existence of a basin of attraction for the origin, we employ Lyapunov functions method. We also provide simple method to detect multistability near the origin based on system reduction to the set of plane curves and studying the possibility of their tangency with small change of parameters.

Point Vortices and Nonlinear Polynomials of the Sawada–Kotera and Kaup–Kupershmidt Equations

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Special polynomials associated with the Painlevé equations and their higher order analogues have been attracting much attention during recent decades. It was shown that these polynomials possess a certain number of interesting properties. For example, their roots form highly regular structures in the complex plane.

In this talk we present the connection between equilibria of point vortices and special polynomials associated with rational solutions of the Sawada-Kotera equation, the Kaup- Kupershmidt equation, their hierarchies, and some other integrable partial differential equations including the Fordy-Gibbons equation.

We obtain that stationary equilibria of point vortices with arbitrary choice of circulations can be described with the help of the Tkachenko equation, while translating relative equilibria of point vortices with arbitrary circulations can be constructed using a generalization of the Tkachenko equation. We prove that roots of any pair of polynomials solving the Tkachenko equation and the generalized Tkachenko equation give positions of point vortices in stationary and translating relative equilibrium accordingly. These results remain valid even if the polynomials inside a pair possesses multiple or common roots.

The synchronization of systems with robust chaos and related communication schemes

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Nowadays the problems related to the information transfer are very important. These are, for example, problems of radio communication, such as the overloading of the frequency range, noises, signal distortion, etc. Moreover, the question of secrecy of the information transmission is also at the focus. One of a possible solution is to use a chaotic communication methods based on the nonlinear admixture of the information signal to the chaotic one (generated by transmitter) and the synchronization of transmitter and receiver – two uni-directionally coupled chaotic generators [A.S. Dmitriev et al. Dynamical chaos: New media for communication systems. M.: Fizmatlit. 2002].

In present work we investigate a communication schemes, which elements (transmitter and receiver) are the generators of robust chaos. The first radio-physically realizable example of such a system is the model of coupled van der Pol oscillators with step-by-step excitation developed recently in [S.P. Kuznetsov // PRL, 95, 2005, 144101]. This system demonstrates chaotic behavior of a hyperbolic type, which is associated with an Smale–Williams attractor. To use the systems with robust chaos, nonsensitive to small disturbances and perturbations is a good choice for technical applications and chaotic communication schemes. In present work the electronic circuits of the communication schemes are developed. The features of synchronization of the robust chaotic generators and functional abilities of communication schemes are studied.

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Quantum-hydrodynamic analogy

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To the memory of
Leonid Pavlovich Shilnikov
Recollecting our discussions

Since papers by quantum theory classics it was paid attention on the analogies between fluid motion and the change of a quantum-mechanics state. E. Madelung marked that from the hydrodynamic continuity equation implies that some function constructed from hydrodynamics values in accordance to some rule obeys the Schrödinger equation. Later P. Debye and D. Bohm introducing a special quantum mechanical potential derived from the Schrödinger equation the equation of potential motion for the ideal fluid.

The second part of XX century was marked by intruding methods and conceptions of quantum mechanics to the classical mechanics of continua. The appearance of the Lax representation and the inverse scattering method has led to integration of classical problems of nonlinear wave theory for the Korteweg-de Vries equation, nonlinear Schrödinger equation and so forth. In this paper developing the analogy between quantum mechanics and hydrodynamics the Lax representation is derived for the Euler equations of the ideal fluid. On its base we obtain a quantum-hydrodynamic analogy that says that the analog of Ψ function (of a quantum state) is some function of Lagrangian coordinates, its value is preserved along the trajectories of fluid particles. Further, in the framework of this analogy there is a conserved quantum-mechanic operator. It is just the special hydrodynamical "vortex operator". Its mean value calculated for any hydrodynamical analog of Ψ function is preserved in time. The existence of specific conservation laws in hydrodynamics (preservation of intensity of vortex tubes, preservation of velocity circulation for liquid contours) corresponds to this phenomenon. As a corollary of preservation of the mean value of "the vortex operator" is as a quantum analog the well known in hydrodynamics Cauchy theorem of the variation of the vortex velocity.

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Bifurcational analysis of extremals of Fredholm functionals with circular symmetry in presence of double resonances

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We consider bifurcational problems of extremals of $SO(2)$-invariant Fredholm functionals at a steady-state point with a double-resonance. Such problems occur in the theory of phase transitions in crystals, in the nonlinear wave theory, also in radiophysics, in economics, in population dynamics, etc. The main result is a list of normal form of key functions (functions has been obtained after nonlinear Ritz approximation of functional) corresponds different resonance types. Also we present examples of the bifurcation analysis of extremals. We construct an approximate algorithm for calculation and analysis of extremals of $SO(2)$-invariant Fredholm functional by the Lyapunov-Schmidt key function.

In the presence of double resonances the key function includes terms of standard invariants of circle and special resonance terms:

$$\frac{1}{2}\left(\sum_{k=1}^{3}\delta_k I_k\right)+\frac{1}{4}\left(\sum_{k=1}^{3}A_k I_k^2+2\sum_{k,j=1}^{3}B_{k,j}I_k I_j\right)+J+o(\|\xi\|^4). \qquad (1)$$

Then we normalize this function using Mather's condition about a finite determinacy of a smooth map germ. It helps us to find a simple representation of the key function. Further bifurcation analysis of branching extremals reduces to analysis of boundary and corner singularities by the secondary reduction.

Rich model cycles in skew-product dynamics: From totally non-hyperbolic dynamics to fully prevalent hyperbolicity via heterodimensional cycles

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Following the model in [Diaz-Horita-Rios-Sambarino ETDS 2009] we introduce a two-parameter family of skew-products $G_{a,t}$, $a \in (0, \infty)$ and $t \in [-\epsilon, \epsilon]$, maps having a heterodimensional cycle (say at $t = 0$). Here a controls the one-dimensional central dynamics and t the unfolding of the cylce. When $a \in (0, \log 2)$ the dynamics is always non-hyperbolic after the unfolding of the cycle. However, for $a > \log 4$ hyperbolic parameter intervals appear, these intervals turn out to be totally prevalent at the bifurcation as a goes to ∞.

The dynamics of these maps is described using a family of iterated function systems modeling the dynamics in the one-dimensional central direction. Properties of these families can be translated to properties of the maps $G_{a,t}$.

This family displays some of the main characteristic properties of the unfolding of heterodimensional cycles as intermingled homoclinic classes of different indices and secondary bifurcations via collision of homoclinic classes.

This is a join work with Esteves (Braganca, Portugal) and Rocha (Oporto, Portugal).

Dynamical chaos for telecommunications

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"There is nothing more practical than a good theory"
Opinion of theorists and not only ...

Dynamical chaos as the fundamental phenomenon had been lucky from the very beginning to fall into interdisciplinary elaboration. This allowed one without any delay to test many results and ideas that were born at the tip of the pen of mathematicians into models of the physical systems and further, into experiments and devices of a practical importance.

The report addresses to the one of the areas where dynamical chaos found the great applications: its use in telecommunication technologies. In some situations, such usage does not impose any specific requirements to the structure of chaos, even to the fact that it has a dynamical nature. In such situations, it is considered merely as an analog noise-like signal. Everything seems to be easy, but where is the chaos? But if we look a little deeper, we find that dynamical systems can create sources of noise-like signals with the energy efficiency by 4-7 times larger than the energy efficiency of the best samples of the noise sources using other principles. Another important feature of chaotic dynamical systems as means to create noise-like signals is the possibility to generate the power spectrum in the desired frequency range and a pre-shaped spectrum. Already the set of only these characteristics determines the unique capabilities of dynamical chaos for telecommunications. No exaggeration is to say that just as the lasers are effective sources of narrow-band light, chaos generators are effective sources of wide-band (UWB) electromagnetic noise-like signals. It is worth noting that the history of "noise-like signals obtained by means of dynamical systems"in radio-electronics began at about the same time as the well known paper by E. Lorenz was published. We present references to two documents which time of appearance is completely definite. In 1965 a patent by C. Reiss et al. "High power noise source employing a feedback path around a travelling wave tube"with a priority of 16 November 1961 was issued. In 1967, V. Ya. Kislov, E.A. Myassine and E.V. Bogdanov obtained a certificate on the "Method of generating electromagnetic noise oscillations"with a priority of 31 July 1967.

Further we discuss the principles of applications of chaotic oscillations in: electronic masking (1974), the protection of information in computer science from eavesdropping on spurious emissions (1981), a short-range radar (works are carried out since the late 70's), electronic warfare (1986), confidential communications (works are conducted since the early 90's), ultra-wide-band wireless personal communications (2000), wireless sensor networks, including medical and multimedia purposes (2004), radio vision (2007), measuring instruments (2013) et al. For the visualization, the report contains a number of rare photos and video materials.

Cascade of Bifurcations of Space-Time Caustics and Wave Trains over Elongated Underwater Banks

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We study the behavior of linear nonstationary shallow water waves generated by an instantaneous localized source as they propagate over and become trapped by elongated underwater banks or ridges. To find the solutions of the corresponding equations, we use an earlier-developed asymptotic approach based on a generalization of Maslov's canonical operator, which provides a relatively simple and efficient analytic-numerical algorithm for the wave field computation. An analysis of simple examples (where the bank and source shapes are given by certain elementary functions) shows that the appearance and dynamics of trapped wave trains is closely related to a cascade of bifurcations of space-time caustics, the bifurcation parameter being the bank length-to-width ratio.

Synchronization in the phase model of three coupled oscillators: from a chain to a ring

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In present work we consider the phase model of a chain of three dissipatively coupled phase oscillators with an additional coupling between the first and third oscillators. The variation of the additional coupling allows us to study how the chain transforms to the system with global coupling. We consider a recurrent map and a system of coupled Adler equations. We distinguish cases of positive coupling parameter (dissipative coupling) and of negative coupling parameter (active coupling). The active coupling can be interesting applied in the field of laser physics [1-3].

The frequency detuning plane is studied by the method of the chart of Lyapunov exponents [4,5]. The transformation of a region of complete synchronization and structure of the regions of quasi-periodical regimes are studied. The last have same structure, as an Arnold resonant web. The transition from a chain to a ring takes place in different ways in the cases of the dissipative and active coupling. We also discussed the transition to chaos with increasing parameter of coupling.

This research was supported by RFBR and DFG grant No.11-02-91334-NNIO.

References

[1] Likhanskii V.A., Napartovich A.P. *Progress physical sciences* **3** 101 (1990). (in Russian)

[2] A.F. Glova. *Quantum Electronics* **4** 283 (2003).

[3] A.F. Glova, A.Y.Lysikov. *Quantum Electronics* **4** 315 (2002).

[4] Baesens C., et al. *Physica D* **49** 387 (1991).

[5] Yu.P. Emelianova, et al. *Physica D* **1** 36 (2013).

On numerical visualization of invariant sets of dynamical systems

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Authors of the talk have developed computer program WInSet for numerical visualization of invariant sets: phase curves, resonance structures, strange attractors, fractals and patterns [1], [2]. An updated version of program WInSet is presented using several examples of systems with homoclinic Poincaré structures which were studied by L.P. Shilnikov.

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References

[1] A.D.Morozov, T.N.Dragunov. Visualization and Analysis of Invariant Sets of Dynamical Systems. – Moscow-Izhevsk: Institute for Computer Studies, 2003.

[2] Morozov A.D. Resonances, cycles and chaos in the quasi-conservative systems. – Moscow-Izhevsk: RCD, 2005.

Lagrangian and Eulerian chaos in confined two-dimensional convection

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We investigate the transition to chaos for the two-dimensional Boussinesq convection inside an air-filled cavity, using accurate numerical time-integration of the Navier-Stokes equations. The fluid is set in motion by an imposed temperature difference between the two lateral walls, proportional to the governing parameter Ra (the Rayleigh number). The problem is considered both from an Eulerian (trajectories in a high-dimensional phase space) and a Lagrangian (trajectories of fluid particles) point of view. In the Eulerian perspective, the system is dissipative and we can track the emergence of attractors as Ra is adiabatically increased. The steady recirculation flow loses its stability through several oscillatory bifurcations $T^0 \to T^1 \to T^2$. As Ra is furthered increased, the system crosses several Arnold tongues and eventually reaches a chaotic attractor. For values of Ra preceding the chaotic regime, interesting attractors with mixed spectra and nonvanishing autocorrelation functions are identified. We will discuss the possible occurence of strange nonchaotic attractors along this route to chaos. In the Lagrangian perspective, the dynamics of the fluid particles is two-dimensional and Hamiltonian. The onset of chaotic advection coincides with the first Hopf bifurcation of the Eulerian system, when the Hamiltonian system loses its integrability. The existence of the main chaotic mixing zones is predicted by the numerical computation of the Melnikov functions associated with each homo/heteroclinic streamline. Multiple additional weakly-mixing zones are created in the vicinity of resonant streamlines as predicted by KAM theory. A finite set of low-period unstable periodic orbits identified numerically is considered along with their stable and unstable manifolds. It is used to establish a coarse-grained cartography of the mixing inside the cavity. This results in a quantification of the mixing area vs. Ra, suggesting that the stable/unstable manifolds of the unstable orbits densely cover the domain before the bifurcation $T^1 \to T^2$ occurs.

This is a joint work with L. Oteski and L. Pastur.

L.P. Shil'nikov's Paper "On the Work of A.G. Maier on Central Motions"and Dynamics of Skew Products

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We describe results of the above paper by L.P. Shil'nikov (*Math. Notes*, **5**:3 (1969), 204-206) in the framework of problems by G. Birkhoff and the papers by A.G. Maier. These works influenced on the appearance results of the talk.
We formulate decomposition theorem for the space $T^1_*(I)$ of C^1-smooth skew products of interval maps such that the quotient map of any map $F \in T^1_*(I)$ has a type $\succ 2^\infty$ and belongs to the space of Ω-stable maps of the interval I_1 into itself (in the space of C^1-smooth maps of the interval I_1 with the invariant boundary) onto the union of 4 nonempty pairwise disjoint subspaces, where $I = I_1 \times I_2$ is a closed rectangle in the plane, I_1, I_2 are closed intervals (Efremova L.S. "On the Space of C^1-Smooth Skew Products of Maps of an Interval", *Theor. and Math. Phys.*, **164**:3 (2010), 1208-1214).
Let $T^1_{*,4}(I)$ be the subspace of the space $T^1_*(I)$, that consists of skew products with the discontinuous Ω-function and the countable set of discontinuous suitable functions for the Ω-function.
It is proved that the maps from the space $T^1_{*,4}(I)$ admit any ordinal number both of the first class and of the second class as the depth of the center, and the cardinality of the set of the Ω-conjugacy classes of maps from $T^1_{*,4}(I)$ is not less than $\aleph_1$, where $\aleph_1$ is the least uncountable cardinality.
Denote by $\widetilde{T}^1_{*,4}(I)$ the subspace of $T^1_{*,4}(I)$ that consists of maps F satisfying: the discontinuity points set of the Ω-function of $F \in \widetilde{T}^1_{*,4}(I)$ is an arbitrary perfect ω-limit set contained a periodic point of the quotient map f. It is proved that the skew products from $\widetilde{T}^1_{*,4}(I)$ with any admissible (i.e. not above the second class) depth of the center, which exceeds any given transfinite ordinal of the second class, are everywhere dense in the space $\widetilde{T}^1_{*,4}(I)$.
Results of this talk demonstrate the impossibility of the complete dynamical description of maps of the space $T^1_{*,4}(I)$ based on the concept of the Ω-conjugacy.

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Effect of broadband synchronization in the systems of two delay-coupled oscillators with cubic-type and Bessel-type nonlinearities

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We study two delay-coupled oscillators with cubic-type nonlinearity of the form

$$\begin{aligned} \frac{dA_1}{dt} + \frac{i\Delta}{2}A_1 + \gamma A_1 &= \alpha_1 e^{i\theta}[(1-k)(1-|A_1^{(t-1)}|^2)A_1^{(t-1)} + \\ &\quad + k(1-|A_2^{(t-1)}|^2)A_2^{(t-1)}], \\ \frac{dA_2}{dt} + \frac{i\Delta}{2}A_2 + \gamma A_2 &= \alpha_2 e^{i\theta}[(1-k)(1-|A_2^{(t-1)}|^2)A_2^{(t-1)} + \\ &\quad + k(1-|A_1^{(t-1)}|^2)A_1^{(t-1)}], \end{aligned} \tag{1}$$

where $A_{1,2}^{(t-1)} = A_{1,2}(t-1)$, and two delay-coupled oscillators with Bessel-type nonlinearity

$$\begin{aligned} \frac{dA_1}{dt} + \frac{i\Delta}{2}A_1 + \gamma A_1 &= 2\alpha_1 e^{i\theta}[(1-k)J_1(|A_1^{(t-1)}|)\frac{A_1^{(t-1)}}{|A_1^{(t-1)}|} + \\ &\quad + kJ_1(|A_2^{(t-1)}|)\frac{A_2^{(t-1)}}{|A_2^{(t-1)}|}], \\ \frac{dA_2}{dt} + \frac{i\Delta}{2}A_2 + \gamma A_2 &= 2\alpha_2 e^{i\theta}[(1-k)J_1(|A_2^{(t-1)}|)\frac{A_2^{(t-1)}}{|A_2^{(t-1)}|} + \\ &\quad + kJ_1(|A_1^{(t-1)}|)\frac{A_1^{(t-1)}}{|A_1^{(t-1)}|}]. \end{aligned} \tag{2}$$

An analytical and numerical investigation of synchronization regimes is performed on different parameter planes. The presence of oscillator death periodic zones is revealed, and the effect of "broadband synchronization which was previously detected in the finite-dimensional systems with non-identical excitation parameters, is observed. Analytical boundaries of the oscillator death region and the broadband synchronization area are in good agreement with the results of numerical investigation of non-identical coupled oscillators. We show that the synchronization picture of two coupled oscillators with Bessel-type nonlinearity (2) does not differ qualitatively from the synchronization picture of two coupled oscillators with cubic-type nonlinearity (1). However, there is a shift of lower boundaries of single-frequency regions, beating and chaotic generation regions to higher values of the excitation parameter (α_1 or α_2) on the parameter plane "coupling value k — excitation parameter". At the same time the range of the excitation parameter values, for which different regimes of generation are realized, increases significantly in comparison with the case of oscillators with cubic-type nonlinearity.

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Effect of the delay time in the coupling channel on synchronization regimes of two multi-mode oscillators

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We study the following system of two coupled oscillators with the delayed feedback:

$$\frac{dA_1}{dt} + \frac{i\Delta}{2}A_1 + \gamma A_1 = \alpha_1 e^{i\theta}[(1-k)\left(1-|A_{1\tau}|^2\right)A_{1\tau} + k\left(1-|A_{2\tau}|^2\right)A_{2\tau}],$$
$$\frac{dA_2}{dt} + \frac{i\Delta}{2}A_2 + \gamma A_2 = \alpha_2 e^{i\theta}[(1-k)\left(1-|A_{2\tau}|^2\right)A_{2\tau} + k\left(1-|A_{1\tau}|^2\right)A_{1\tau}]. \qquad (1)$$

Here, $A_{1\tau} = A_1(t-\tau)$, $A_{2\tau} = A_2(t-\tau)$. α_1 and α_2 are the excitation parameters of the first and the second oscillators, respectively. γ represents the dissipation parameter, and θ is the phase shift in the feedback loop. k denotes the coupling parameter, which characterizes the ratio of powers in the feedback circuits ($0 < k < 1$). Δ is the frequency detuning, and τ is the delay time, associated with the finite time of the signal passage through the feedback circuit, i.e. with the length of the feedback channel.

The feature of the synchronization picture of coupled oscillators (1) is an existence of oscillator death periodic zones separated from the beating region by a narrow band of synchronous regimes on the parameter plane "frequency detuning Δ — coupling parameter k". This periodicity is caused by the resonances between different eigenmodes of oscillators. The numerical and analytical investigation reveals the effect of the delay time on the synchronization picture of oscillators. In particular, with the increase of the delay time a number of oscillators' modes increases. This leads to the decrease of the thickness of the periodic band of synchronous regimes, and to the more frequent arrangement of the oscillator death periodic zones. Moreover, an increase of the delay time leads to the same effects as an increase of the excitation parameters of oscillators: a) a lower boundary of the periodic band of synchronous regimes shifts to higher values of the coupling parameter; b) the appearance of the chaotic generation regimes occurs at lower values of the excitation parameters.

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Orbits of intervals for one-dimensional dynamical systems

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The necessity of considering in the phase space orbits of subsets other than points occurs both in dynamical systems theory by itself and in many evolutionary problems reducible to dynamical systems.

There are classes of dynamical systems for which the asymptotic behavior of orbits of subsets of the phase space is similar to the behavior orbits points. For example, if the global attractor of the dynamical system is a fixed point, orbits of any subset in the phase space, as well as the trajectory of any point of the system tend to this fixed point. On the other hand, there are classes of dynamical systems for which the behavior of orbits of subsets is completely different from the behavior of orbits of points. For example, orbits of points for one-side Bernoulli shift have chaotic behavior, but the orbit of any cylinder (as subset of phase space for this dynamical system) has very simple regular behavior, moreover, it takes only a finite set of values.

In the first part of the talk the dynamical systems generated by continuous interval maps are considered. For such systems a series of results is presented on asymptotic periodicity of orbits of subintervals [1]. In the second part we discuss the problem on asymptotic behavior of orbits of subsets for some other classes of dynamical systems which study is reduced to interval maps, namely, first order difference equations with continuous time, many-sheeted and hybrid dynamical systems generated by linear ordinary second order differential equations [2].

References

[1] Fedorenko V. V. *Asymptotic periodicity of trajectories of intervals*, Ukrainian Math. J. *61* (2009), 416–420 [in Russian].

[2] Fedorenko V. V. *Trajectories of domains of some low-dimensional dynamical systems*, Grazer Math. Ber. *354* (2009), 37–45.

The effect of weak nonlinear dissipation on the stochastic web

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It is known [1] that a degenerate Hamiltonian systems (in the sense of KAM theorem) demonstrates a special structure of the phase space called the stochastic web.

The effect of weak nonlinear dissipation on the structure of the phase space of such systems is investigated in our work. We consider the driven linear oscillator with the small dissipative perturbation :

$$\ddot{x} + (\gamma - \mu x^2)\dot{x} + \omega_0^2 x = -\frac{\omega_0 K}{T}\sin(x+\varphi)\sum_{n=-\infty}^{+\infty}\delta(t-nT).$$

So, the autonomous system is van der Pol generator which demonstrates the limit cycle if the parameters of the linear γ and nonlinear dissipation μ are negative. Another parameter is the ratio of pulse period to the own period of the system $q = 2\pi/\omega_0 T$, which determines the type of symmetry of the web [1].

The evolution of attractors of the stroboscopic map of system with the increase of the nonlinear dissipation μ for fixed small values of the linear dissipation γ was studied.

Attractors are steady foci at small μ. With the increase of nonlinear dissipation two neighboring attractors merge with the saddle via the pitchfork bifurcation and then the formed stable node disappears via saddle-node bifurcation. Also, with the increase of μ stable invariant curve appears in the result of the non-local bifurcation of saddle points manifolds. With essential μ this invariant curve remains the only attractor in the system which corresponds to the quasi-periodic oscillations. The only exception is the case of $q = 3$ when the result is the invariant curve or the stable focus depending on the parameter φ.

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References

[1] G.M. Zaslavsky. Physics of Chaos in Hamiltonian Systems. Imperial College Press, 1998

Breaking of Ergodicity in Expanding Coupled Map Lattices

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To identify and to explain phase transitions in Coupled Map Lattices (CML) has been a lingering enigma for about two decades. In numerical simulations of standard models, this phenomenon has always been observed preceded by a lowering of the Lyapunov dimension, suggesting that the transition might require a stability change of the attractor. Yet, recent proofs of co-existence of several phases in specially designed CML (inspired by PCA) work in the expanding regime where all Lyapunov exponents must be positive.

In this talk, I will consider a family of CML with piecewise expanding individual map, global interaction and finite number of sites, in the weak coupling regime where the CML is uniformly expanding. I will show that a transition in the asymptotic dynamics occurs as the coupling strength increases. The transition breaks the attractor into several chaotic pieces of positive Lebesgue measure, with distinct empiric averages. It goes along with simultaneous breaking of various symmetries, which can be quantified by measuring the amplitude of magnetization-type characteristics.

Despite that it only addresses finite-dimensional systems, to some extend, this result reconciles previous ones as it shows that a loss of ergodicity (associated with symmetry breaking) can occur in basic CML, independently of any decay in the Lyapunov dimension.

Transport barriers in time-dependent flows and their consequences for pattern formation

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Oceanic flows contain meso-scale structures like vortices and jets which play an important role in the transport of different substances or small organisms like plankton which are mainly passively transported. Besides those structures stagnation points of the fluid act as organizing centers of the flow, since particles approach them along certain directions and move away from them in other directions. If the fluid could be described as an Hamiltonian system vortices would correspond to KAM tori while the stagnation points, at which the velocity vanishes, would represent fixed points of saddle type possessing stable and unstable manifolds. Since natural flows are in general aperiodic, these structures exist only for a certain time span and move in space. To use concepts from dynamical systems theory to analyze the topology of the flow and to quantify transport one has to extend those concepts to aperiodic flows. Two such concepts are widely used in hydrodynamic flows: finite size Lyapunov exponents (FSLE) or finite time Lyapunov exponents (FTLE) as well as Lagrangian coherent structurs (LHS) and distinguished hyperbolic trajectories (DHT). These quantities are used to identify transport barriers in flows. For marine ecosystems the transport of nutrients is one of the essential determinants for plankton blooms, particularly such blooms which are harmful to other organisms or even humans. We detect transport barriers in kinematic two-dimensional idealized flows as well as in real ocean flows by means of the methods mentioned above and study their consequences for plankton blooms and bio-diversity.

ODE meanders and PDE global attractors: a Sturmian view

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For scalar parabolic equations in one space dimension there are many results which relate the equilibrium ODE

$$v'' + f(x, v, v') = 0$$

with the global structure of the PDE attractor of the associated semilinear parabolic ODE, say under Neumann boundary conditions on the unit interval. A central notion is the meander permutation introduced by Fusco and Rocha in 1991.

If f is 1-periodic in x, transverse homoclinic orbits may arise for the above "pendulum". On intervals $0 < x < L$ of large integer length L, these homoclinics strongly influence the behavior of the associated meander permutation, and hence of the PDE attractor. Although the precise effect remains elusive, at present, we attempt to illustrate the problem in the more elementary case of standard Anosov diffeomorphisms on the 2-torus.

Boundary Conditions for Fiber Maps and the Topological Transitivity of Skew Products of Interval Maps

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The sufficient conditions (theorem A) and the criterion (theorem B) are formulated for the topological transitivity, but not the topological ergodicity of maps from the class of C^3-smooth skew products of interval maps with the noninvariant boundary of fibers.
Let $T^3_{fb}(I)$ ($I = [a_1, b_1] \times [a_2, b_2]$) be the space of C^3-smooth skew products of interval maps $F(x,\, y) = (f(x),\, g_x(y))$, $F : I \to I$, satisfying:
$(C.1)$ Schwarz's derivative of any fiber map $g_x(y)$ ($x \in [a_1, b_1]$) is negative for all $(x, y) \in I$ such that $\frac{\partial}{\partial y} g_x(y) \neq 0$;

$(C.2)$ there is the unique critical point of a map $g_x(y)$ in the interval (a_2, b_2) for any $x \in [a_1, b_1]$, and this point is nondegenerate;
$(C.3)$ $a_2 < g_x(a_2) \le b_2$ and $g_x\,(b_2) = a_2$ for all $x \in [a_1, b_1]$.

Theorem A. *Let $F \in T^3_{fb}(I)$ satisfy the following conditions:* $(Y.1)$ *the equalities $p(x) = p(f(x)) = p(f^2(x)) = \ldots = p(f^{n-1}(x))$ are valid for all $x \in Per(f)$, where $p(x)$ is the unique fixed point of the map $g_x(y)$, $Per(f)$ is the set of f-periodic points, n is the (least) period of x;* $(Y.2)$ *$p(x)$ is the source of the map $g_x(y)$ for any $x \in [a_1, b_1]$;*
$(Y.3)$ *$f(x)$ is the topologically ergodic;* $(Y.4)$ *$g_x(a_2) = p(x)$ for any $x \in Per(f)$.*
Then F is the topologically transitive but not the topologically ergodic skew product.
The points $\{p(x)\}_{x\in[a_1,\,b_1]}$ form the segment $[a_1, b_1] \times \{p\}$, that separates I on two nonempty closed subrectangles R_1 and R_2.

Theorem B. *Let $F \in T^3_{fb}(I)$ satisfy conditions* $(Y.1)$, $(Y.3)$, $(Y.4)$. *Then F is the topologically transitive but not the topologically ergodic skew product iff I is uniformly approximated by F-periodic orbits with even periods; moreover, F^2-periodic orbits of points from $Per^*(F) \cap R_1$ ($Per^*(F) \cap R_2$) uniformly approximate the rectangle R_1 (R_2), where $Per^*(F)$ is the set of periodic points with even periods such that their orbits uniformly approximate I.*

This is the joint work with L. S. Efremova.

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Allee effect and dynamics behaviour in Richards' growth models

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Population dynamics have been attracting interest since many years. Among the considered models, the Richards equation remains one of the most popular to describe biological growth processes. The Allee effect is currently a major focus of ecological research, which occurs when positive density-dependence dominates at low densities. Its main interest concerns the population management of rare species. In this talk we propose the dynamical study of classes of functions based on Richards' models describing the existence or not of Allee effect. Subclasses of strong and weak Allee functions and functions with no Allee effect are characterized. The study of the bifurcation structure in parameter space is presented, this analysis is done by considering bifurcation curves and symbolic dynamics techniques. Generically, the dynamics of these functions are classified in the following types: extinction, semi-stability, stability, bifurcations, chaos, chaotic semistability and essential extinction. We obtain conditions on parameter values for the existence of a weak Allee effect region related to the appearance of cusp points. To support our results, we present bifurcations curves and numerical simulations of several bifurcation diagrams.

This is a joint work with J.L. Rocha (ISEL, ADM and CEAUL Lisboa, Portugal) and A.K. Taha (INSA Toulouse, France).

Critical homoclinic orbits, appearance of snap-back repellers and Ω-explosions in piecewise smooth maps

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When nondegenerate homoclinic orbits to an expanding fixed point of a map $f : X \to X, \quad X \subset R^n, \quad n > 0$ exist, this fixed point is called a *snap-back repeller*. It is known that the presence of a snap-back repeller implies the existence of an invariant set on which the map is chaotic. Therefore the following question arises: When does the first homoclinic orbit appear, causing an expanding fixed point to become a snap-back repeller? Another question, which turns out to be closely related to the previous one is: When can other homoclinic explosions, i.e., appearance of infinitely many new homoclinic orbits, occur?

In this talk we extend the results regarding these bifurcations previousely known for smooth maps to a more broad class of maps (smooth or piecewise smooth, continuous or discontinuous, defined in a bounded or unbounded closed set). We distinguish between *critical* and *noncritical* homoclinic orbits and discuss briefly the possible kinds of critical homoclinic orbits. We demonstrate that only critical homoclinic orbits of an expanding fixed point are responsible for the appearance of snap-back repellers as well as for other the homoclinic explosions. We show also that a homoclinic orbit of an expanding fixed point is structurally stable if and only if it is noncritical.

Asymptotics beyond all orders near a Hamiltonian bifurcation

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In this talk we discuss recent results obtained in collaboration with L. Lerman on non-existence of a single-bump orbit homoclinic to an equilibrium in a Hamiltonian system near a ghost separatrix loop created in a generic unfolding of a $(0^2, \pm i\omega)$ Hamiltonian equilibrium.

On structure and reduction in some Hamiltonian homoclinic bifurcation problems

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We examine the inheritance of Hamiltonian structure in reduced bifurcation equations obtained via the Lyapunov-Schmidt method for homoclinic orbits. This functional analytic approach is motivated by the study of travelling wave equations in lattice differential equations, which lead in general to ill-posed initial value problems, and are thus difficult to study via other methods based on flow properties. We are mostly interested in the case of a homoclinic loop to a nonhyperbolic equilibrium; we examine the configuration of accompanying homoclinic orbits to the local center manifold of the equilibrium.

Modelling of the influence of zooplankton activity on the ecosystem state

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The paper deals with the interaction of plankton populations in a vertical column of water. At modeling of the vertical distribution of zooplankton is necessary to consider not only the passive movement, but also active movement of organisms. They can have various causes, including seasonal, age, daily, associated with the need for reproduction, nutrition, protection from enemies, due to hydrological conditions.

The system is described by a three-component model of the interaction of the plankton community with the nutrients. Dynamics of nutrients, phytoplankton and zooplankton is considered in the upper mixed layer of water. Movement of matter and organisms in depth is described by the turbulent diffusion. Directional movement of zooplankton is described by the advective terms: diel migration and taxis.

Local change of nutrients concentration is due to inflow of nutrients in the treated area and the outflow, remineralization of dead organic matter and participate in photosynthesis. The increase in phytoplankton biomass is in the process of photosynthesis and the decrease - as a result of grazing by zooplankton, natural mortality and leaching from the treated area. Zooplankton biomass increases due to trophic interactions with phytoplankton and decreases due to natural mortality, intraspecific competition and the washout.

As shown by the model calculations, the diel vertical migrations are making significant changes in the behavior of plankton only in conjunction with the search activity (taxis) and food regime. Spatial moving increases the population of zooplankton, but reduce the population of phytoplankton. Inclusion of these two variants of activity does not lead to over-exploitation phytoplankton only when zooplankton have a nocturnal grazing regime.

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Bogdanov-Takens bifurcation in delay differential equation with asymptotically large period cycle appearance

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We consider mathematical model that is based on delay differential equation [1]

$$\frac{dy}{dt} = a\,[\,y - y(t{-}1)] - f(y), \qquad (1)$$

where a is real parameter, $y(t)$ is scalar function. Nonlinearity is $f(y){=}|y|^{n-1}y$.

Stability of the solution $y{=}0$ is defined by $\lambda{=}a[1{-}e^{-\lambda}]$. Anyway it has the root $\lambda{=}0$, that becomes double while $a{=}1$. This situation corresponds to so-called Bogdanov-Takens bifurcation. If $a{<}1$ then zero solution of (1) is stable and it is unstable if $a{>}1$.

We use normal forms method for study equation (1) features in the vicinity of critical parameter value $a{=}1{+}\,\varepsilon$. Substituting

$$y(t,\varepsilon) = \varepsilon^{1/n}\Big(x(s) + \varepsilon^{1/2}\dot{x}(s) + \ldots\Big), \qquad s = \varepsilon^{1/2}t, \qquad (2)$$

we obtain [2] ODE for function $x(s)$, where $f\,'(y){=}n|y|^{n-1}$ (for $n{\geq}2$):

$$\ddot{x} + 2\sqrt{\varepsilon}\,\Big[f\,'(x)/3 - 1\Big]\dot{x} + 2\,f(x) = 0\,. \qquad (3)$$

Theorem. *Equation (3) has the only periodic solution that is asymptotically orbitally stable while $s{\to}\infty$.*

Limit cycle existence is proved by second method of Lyapunov. Uniqueness and stability is justified by method of Pontryagin. It also allow us to calculate all characteristics of the cycle. Substitution (2) confirms numerical result that period of oscillations in equation (1) tends to infinity as $\varepsilon^{-1/2}$ and amplitude tends to zero as $\varepsilon^{1/n}$ while $\varepsilon{\to}{+}0$.

References

[1] *Erneux T., Walther H.O.* Bifurcation to large period oscillations in physical systems controlled by delay // Phys. Rev. E. — 2005. — Vol. 72, 066206.

[2] *Glazkov D.V.* Normalization of delay differential equation and bifurcation leading to an asymptotically large period cycle // Modeling and analysis of information systems. — 2007. — V. 14, N2. — P. 47-52 (in Russian).

Relaxation oscillations in neuro-dynamic systems with delays

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We consider a singularly perturbed scalar nonlinear differential-difference equation with two delays that models the electrical activity of a neuron

$$\dot{u} = \lambda[f(u(t-h)) - g(u(t-1))]u. \tag{1}$$

Here, $u(t) > 0$ is the membrane potential of the neuron, $\lambda > 0$ is a large parameter, and $0 < h < 1$ is a fixed delay. The functions $f(u),\ g(u) \in C^2(R_+),\ R_+ = \{u \in R : u \geq 0\}$, are assumed to have the following properties: $f(0) = 1,\ \ g(0) = 0;\ \ f(u) = -a_0 + O(1/u),\ \ uf'(u) = O(1/u),\ \ u^2 f''(u) = O(1/u),\ g(u) = b_0 + O(1/u),\ ug'(u) = O(1/u),\ \ u^2 g''(u) = O(1/u)$ as $u \to +\infty$, where a_0 and b_0 are positive constants.

It is well known that self-excited oscillations in neural systems exhibit two characteristic features, namely, the bursting effect and the buffer phenomenon. The former is characterized by series of intensive peaks alternating with relatively quiet segments of membrane potential variations. The latter is said to occur when any prescribed finite number of coexisting attractors are observed in a dynamical system. As a rule, the mathematical modeling of the bursting effect is based on singularly perturbed systems of ordinary differential equations with one slow and two fast variables. Another approach is associated with allowance for time delays.

Now consider a one-dimensional chain of m neurons (Eq. (1)), $m \geq 2$, each interacting with two nearest neighbors. In this case, Eq. (1) is replaced with the system

$$\dot{u}_j = d\,(u_{j+1} - 2u_j + u_{j-1}) + \lambda[f(u_j(t-h)) - g(u_j(t-1))]u_j,\ \ j = 1, \ldots, m, \tag{2}$$

where $u_0 = u_1$, $u_{m+1} = u_m$, and $d > 0$ is a parameter of order 1 characterizing the depth of the interaction between the neurons.

It is shown that, with a suitable choice of parameters, any prescribed finite number of stable bursting cycles can coexist in the phase space of this system.

On existence of Lorenz-like attractors in a nonholonomic model of celtic stone

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We consider a nonholonomic model of celtic stone movement along the plane. As is well-known, the movement of celtic stone on the plane is still considered as one of most complicated and badly studied type of the rigid body movement. Moreover, it is one of the types of such movements where chaotic dynamics is possible.

The existence of strange attractors in the celtic stone dynamics was recently shown in [1]. In [2] these results were extended and main bifurcations leading to chaos appearance were studied. In particular, various types of chaotic dynamics were found in the model: a spiral strange attractor, torus-chaos attractors, nearly conservative chaos and even the so-called mixed dynamics [3]. The latter type of a chaotic orbit behavior means that the corresponding non-wandering set contains infinitely many coexisting periodic orbits of all possible types: stable, completely unstable, saddle and, due to reversibility of the system, symmetric elliptic periodic orbits. Moreover, for certain types of celtic stones (possessing certain geometrical and physical properties) their nonholonomic models demonstrate the presence of strange Lorenz-like attractors. In this talk we overview the results related to scenarios of the appearance and breakdown of such strange attractors.

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References

[1] Borisov A.V., Mamaev I.S. *Strange attarctors in dynamics of celtic stones* // Advances in Physical Sciences, 2003, v.117, No., 407-418.

[2] Gonchenko A.S., Gonchenko S.V., Kazakov A.O. *On new aspects of chaotic dynamics of "celtic stone"* // Rus. Nonlin. Dyn., 2012, v.8, No.3, 507-518.

[3] Gonchenko S.V., Turaev D.V. and Shilnikov L.P. *On Newhouse domains of two-dimensional diffeomorphisms with a structurally unstable heteroclinic cycle* // Proc. Steklov Inst. Math., 1997, v.216, 70-118.

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On universal scenarios of chaos appearance for three-dimensional diffeomorphisms

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The problem of chaos appearance for three-dimensional diffeomorphisms is discussed. We propose two new universal scenarios of strange attractors (SA) emergence starting from a stable fixed point to a strange homoclinic attractor, see [1] for more details. In both scenarios, the fixed point loses stability and becomes saddle and we say that appearing SA is *homoclinic* if it contains this saddle fixed point along with its unstable invariant manifold.

For the first scenario, the stable fixed point loses stability at a torus birth (Andronov-Hopf for maps) bifurcation (a pair of multipliers $e^{\pm i\varphi}$, where $\varphi \neq 0, \pi, \pi/2, 2\pi/3$, appears): the fixed point becomes a saddle-focus and a stable smooth closed invariant curve is born. The coming homoclinic attractor will be of spiral type: it contains a saddle-focus with two-dimensional unstable manifold. SA of such type are called also as "Shilnikov-like SA" or "screw SA".

For the second scenario, the stable fixed point loses stability under a period doubling bifurcation: the fixed point becomes a saddle and a stable period 2 orbit is born. The saddle fixed point will have the negative unstable multiplier and two real stable multipliers of opposite sings. The coming homoclinic attractor will be a Lorenz-like SA when the negative stable multiplier is strong stable or a "figure-eight" SA (a Hénon-like SA for strongly dissipative maps) when the positive stable multiplier is strong stable.

We give a qualitative description of these attractors and define certain conditions when they can be *genuine* ones (pseudo-hyperbolic strange attractors). We present also the corresponding results of a numerical analysis of attractors for three-dimensional quadratic (Hénon) maps.

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References

[1] Gonchenko A.S., Gonchenko S.V., Shilnikov L.P. Towards scenarios of chaos appearance in three-dimensional maps // Rus. Nonlinear Dynamics, 2012, v.8, No.1, pp.3-28

On Lorenz-like attractors birth under global bifurcations

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We discuss the problem of Lorenz-like attractors birth under bifurcations of three-dimensional diffeomorphisms with (a) quadratic homoclinic tangencies and (b) with nontransversal heteroclinic cycles. In the paper [1] such a problem has been solved for the case when the initial diffeomorphism has a homoclinic tangency to a saddle-focus fixed point of type (2,1), i.e. with two-dimensional stable and one-dimensional unstable manifolds, and which is conservative-like, i.e. the Jacobian of the diffeomorphism in this point is equal to 1. In [2,3] analogous problems were considered for heteroclinic cycles containing two fixed points one of which, [2], or both, [3], are saddle-foci. Moreover, it was assumed in [2,3] that the diffeomorphism is "contracting-expanding", i.e. the Jacobian is less than 1 at one fixed point and greater than 1 at another point. In this talk, we take main attention to the cases (a) when the fixed point is a saddle, again conservative-like and type of (2.1) and (b) the "contracting-expanding" diffeomorphism has a nontransversal heteroclinic cycle containing two fixed points which are both saddles.

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[1] Gonchenko S.V., Meiss J.D., Ovsyannikov I.I. Chaotic dynamics of three-dimensional Henon maps that originate from a homoclinic bifurcation // Regular Chaotic Dyn., 2006, v.11, p.191–212.

[2] Gonchenko S.V., Shilnikov L.P., Turaev D. On global bifurcations in three-dimensional diffeomorphisms leading to wild Lorenz-like attractors // Regular and Chaotic Dynamics, 2009, v.14, No.1, p.137–147.

[3] Gonchenko S.V., Ovsyannikov I.I. On bifurcations of three-dimensional diffeomorphisms having a non-rough contour containing saddle-

foci (in Russian) // Rus. Nonlinear Dynamics, 2010, v.6, No 1, p. 61-77.

On dynamical properties of area-preserving maps with homoclinic tangencies

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We use qualitative and topological methods to study the orbit behavior near nontrans-verse homoclinic orbits in area-preserving maps which are not necessarily orientable. Let f_0 be such C^r-smooth map ($r \geq 3$) having a saddle fixed point O whose stable and unstable invariant manifolds have a quadratic or cubic tangency at the points of some homoclinic orbit Γ_0. Let f_ε be a family (unfolding) of area-preserving maps containing the map f_0 at $\varepsilon = 0$.

Our aim is to study bifurcations of the so-called single-round periodic points in the family f_ε. Every point of such an orbit can be considered as a fixed point of the corresponding *first return map*. We study bifurcations of the fixed points and in the case of a quadratic homoclinic tangency we prove the existence of cascades of generic elliptic periodic points for one and two parameter unfoldings f_ε. Thus, we generalize the results obtained in [?] where only the symplectic (area-preserving and orientable) case was analyzed. In the case of a cubic homoclinic tangency we establish the structure of bifurcational diagram in two parameter unfoldings f_ε.

References

[1] M. Gonchenko, S. Gonchenko, *On cascades of elliptic periodic points in two-dimensio-nal symplectic maps with homoclinic tangencies*, Regular and Chaotic Dynamics **14(1)**, 2009. 116-136.

On dynamics of diffeomorphisms with hyperbolic nonwandering sets on 3-manifolds

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We present an overview of recent results on the dynamics of diffeomorphisms in dimension 3 obtained by the author in collaboration with Russian (V. Medvedev, O. Pochinka, E. Zhuzhoma) and French (C. Bonatti, F. Laudenbach) mathematicians. We will discuss the problems of topological classification for diffeomorphisms with finite and infinite hyperbolic nonwandering set, interrelations between dynamics and topology of an ambient manifold, existence of global Lyapunov function for a diffeomorphism whose set of critical points coincides with nonwandering set (such function is called to be the energy function) and existence of a simple arc connecting two Morse-Smale diffeomorphisms on 3-manifold. In more details all this can be found in [1],[2].

The author thanks RFBR 12-01-00672-a for the financial support.

References.

[1] V. Z. Grines, O.V. Pochinka. Morse-Smale cascades on-manifolds. Uspehi matematicheskih nauk, 68:1(409) (2013), 129–188.

[2] V. Z. Grines, O.V. Pochinka. Introduction to the topological classification of cascades on manifolds of dimension two and three. "Regular and Chaotic Dynamics"Research Center; Izhevsk Institute for computer Studies, Moscow, Izhevsk 2011, 424 pp.

Overview of the theoretical research and experimental realizations one-dimensional many-body almost-integrable systems with cold quantum gases

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Recent experimental advances in cooling and trapping neutral atoms have given possibility to implement an almost perfect realization of an isolated many-body quantum integrable system—a one-dimensional Bose-Einstein quasicondensate. In my talk I would like to outline the most recent theoretical and experimental research on such an almost-integrable system, including equilibration to the stationary state (presumably described by the Generalized Gibbs ensemble), emergence of the temperature, and the effects of weak integrability breaking which may elucidate the possible generalizations of the KAM theorem to infinite-dimensional Hamiltonian systems.

Construction algorithms for the dynamic mode maps of the nonlinear dynamic systems

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The important thing in studying the nonlinear systems is computer modeling and, in particular, a construction of the dynamic mode maps. That allows one to make a more detailed study of the system, to model and visualize the dynamics, construct the attractors, compute the Lyapunov indices, make the bifurcation analysis, compute the dimension characteristics, etc. We present discussions of algorithms for two-parametric bifurcation analysis that allow us to construct the dynamic mode maps in neighborhoods of the non-hyperbolic equilibrium points or cycles.

The main object of studying is the dynamic system which depends on two scalar parameters α and β and is described by the differential equation

$$x' = A(\alpha, \beta, t)x + a(t, \alpha, \beta, x), x \in \mathbb{R}^2 \qquad (1)$$

where the matrix $A(\alpha, \beta, t)$ and the nonlinearity $a(t, \alpha, \beta, x)$ are T-periodic in t and continuously differentiable in all its arguments, herewith the nonlinearity $a(t, \alpha, \beta, x)$ contains terms of the order 2 and higher in x. Assume that matrix $A(0, 0, t)$ has purely imaginary Floquet indices $\pm i\omega_0$. In this case, for values of (α, β) being close to $(0, 0)$, the emergence of quasi-periodic and periodic solutions in a neighborhood of the equilibrium point $x = 0$ in the system (1) is possible. Here we study the problem of an approximate construction of the dynamic mode maps for the system (1).

The expected research scheme is based on the transition from (1) to the system

$$x_{n+1} = (1 + \alpha)Q(\beta)x_n + b(x, \alpha, \beta), \qquad (2)$$

where the orthogonal matrix $Q(\beta)$ is constructed from the matrix $A(\alpha, \beta, t)$. The cycles of the system (2) determine the periodic solutions of the equation (1). Further, based on the small parameter method, we construct the approximate formulae for the cycles of the system (2) and the corresponding values of the parameters. These formulae are basic for the construction of the dynamic mode maps of the system (1).

On topological classification of Morse-Smale diffeomorphisms on a sphere of dimension four and higher

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We solve a problem on topological classification in class G of orientation preserving Morse-Smale diffeomorphisms without heteroclinic intersections given on the sphere S^n of dimension $n \geq 4$. Such diffeomorphisms have finite non-wandering set consisting of hyperbolic periodic points whose invariant manifolds do not intersect. Topological classification for similar flows was obtained [1] using the classical approach taking its origin in works of A.A. Andronov, L.S. Pontryagin, E.A. Leontovich and A.G. Maier.

Despite the apparent similarity with the flow case, diffeomorphisms from G may exhibit more complex dynamics due to non-trivial embedding of invariant manifolds for saddle periodic points. To describe this embedding we adopt technique of paper [2] and relate to each diffeomorphism $f \in G$ a global scheme S_f which is a set of wandering orbits with projections of $(n-1)$-dimensional invariant manifolds.

We introduce a definition of equivalence of two schemes and prove that equivalence of schemes S_f, S'_f is a necessary and sufficient condition for f, f' to be topologically conjugated.

Results under presentation were obtained in collaboration with V. Grines, and O. Pochinka.

Research is supported by grants 12-01-00672 and 11-01-12056-ofi-m of RFFR.

References

[1] Пилюгин С.Ю. Фазовые диаграммы, определяющие системы

Морса-Смейла без периодических траекторий на сферах// Дифференциальные уравнения, 1978.- Т. 14, № 2. - С. 245-254.

[2] Bonatti Ch., Grines V., Pecou E. Two-dimensional links and diffeomorphisms on 3-manifolds// Ergod. Theory Dyn. Syst., 2002.-v. 22, №. 3 – P. 687–710.

Description of cycles for quadratic maps from the position of the linear conjugacy

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We consider the difference equation $x_{n+1} = f(x_n)$, $n = 0, 1, 2 \ldots$ with real quadratic function $f(x) = ax^2 + b\,x + c$ $(a \neq 0)$ or, briefly, the quadratic map f. It is known that quadratic maps demonstrate a great variety of different types of orbits, both periodic (cycles) and chaotic. In their study various methods are used: symbolic dynamics, ergodic theory and combinatorial methods. When analyzing quadratic maps, however, can be useful the simple considerions related to the concept of linear conjugacy considered in the report. We divide the family A of quadratic maps into the classes of equivalence A_u relative the linear conjugacy, here $u = (b-1)^2 - 4ac$.

Important characteristics of quadratic maps are the followings invariants w.r.t. linear conjugacy: number of k-cycles (k is the period cycle), multiplier μ_k of k-cycles. For $k = 1$ and $k = 2$ have simple formulas $\mu_1(u) = 1 \pm \sqrt{u}\,(u \geqslant 0)$, $\mu_2(u) = 5 - u\,(u > 4)$. Let map $f \in A_u$ have a k-cycle $C_k = (x_0, x_1, \ldots, x_{k-1})$, $k > 2$. Then the normalized k-cycle $N_k = (0, z_1, \ldots, z_{k-2}, 1)$ has some map class A_u. Here z_i – are marks characterizing the type of cycle C_k.

On the plane of (z_1, z_2) algebraic curves G_k are determined in such a way that for $(z_1, z_2) \in G_k$ every map $f \in A_{u(z_1,z_2)}$ has a k-cycle with a chosen value of marks of z_1, z_2; therefore the function of $u(z_1, z_2)$ is defined. The curve G_3 is located on the line $z_2 = 1$. At $k > 3$ the branches the curves G_k, corresponding different k-cycles, are selected. This construction allows one to probe bifurcations of k-cycles, for every k-cycle determine dependence of multiplier μ_k on u, to expose the situations when k-cycles are stable. Basic difficulty consists in the selection of branches for curves G_k, it is aggravated when k is growing.

Similar technique author used for families of cubic maps: Вестн. Удмуртск. ун-та. Матем. Мех. Компьют. науки, 2011, № 1, 20-39.

US Office of Naval Research Global International Science Program

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The US Office of Naval Research (ONR) Global International Science Program employs technically skilled scientists and engineers to enhance the international science and technology (S&T) engagement of the Navy and to increase the Navy's awareness of global technology. The technical staff, known as associate directors are typically scientists with doctorates working across government, academia, and industry. They work out of offices around the world to scout technologies for the Office of Naval Research and the Naval Research Enterprise (NRE). This presentation will outline key opportunities that are available to international scientists for research support and include:

1. Liaison Visits - ONR Global Associate Directors make visits to academic institutions and industry partners to promote collaboration with international scientists and to find cutting edge science and technology.

2. The Visiting Scientist Program (VSP) supports travel of international scientists to discuss new S& T ideas and discoveries with the NRE. The VSP is a vehicle for planning international research programs through information exchange, discussion of S&T advances and the development of long-term relationships between the Department of the Navy and international researchers.

3. The Collaborative Science Program (CSP) provides financial assistance for international engagements of naval interest. CSP funds are intended to support collaborative forums which make a clear contribution to the advancement of NRE S&T programs.

4. The Naval International Cooperative Opportunities in Science and Technology Program (NICOP) is a mechanism to encourage international science and technology cooperation in areas of interest to the Naval

Research Enterprise (NRE) by providing seed funding in collaboration with the Office of Naval Research (ONR) headquarters or other organizations within the NRE. The key to a successful NICOP submission is pulling together a team of international and U.S. scientists to address problems of naval S& T interest. A NICOP provides direct research support to international scientists to help address naval S&T challenges. NICOPs support transformational R&D initiatives of the Navy and Marine Corps; promote the usage of emerging technologies; and accelerate the introduction of innovative ideas into the NRE.

A brief overview of the Irregular and Expeditionary Warfare portfolio will be provided which outlines efforts towards enabling Naval forces to identify, anticipate, preempt, and defeat adaptive irregular threats operating within the complex physical, cyber, and socio-cultural domains with a focus on the Sciences Addressing Asymmetric and Explosive Threats Program.

Synchronization in iterated function systems

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We treat synchronization for iterated function systems on compact manifolds in two cases:

1. iterated function systems generated by random diffeomorphisms with absolutely continuous parametric noise,

2. iterated function systems generated by finitely many diffeomorphisms.

Synchronization is the convergence of orbits starting at different initial conditions when iterated by the same sequence of diffeomorphisms. The iterated function systems admit a description as skew product systems of diffeomorphisms on compact manifolds driven by shift operators. Under open conditions including transitivity and negative fiber Lyapunov exponents, we prove the existence of a unique attracting invariant graph for the skew product system. This explains the occurrence of synchronization.

New types of attractors

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New locally generic properties of attractors will be considered: invisibility, Lyapunov instability, thick and bony attractors. Special ergodic theorems and Holder properties of central fibers of partially hyperbolic systems will be discussed. These are joint works with many coauthors that will be named.

Bifurcation mechanisms of the birth and destruction of hyperbolic chaos

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In the theory of dynamical chaos the so-called hyperbolic strange attractors are known. The behavior on these invariant sets is strongly chaotic and possesses the robustness – non-sensitivity to small perturbations and noises. Hyperbolic strange sets up to recent times have been referred as artificial mathematical objects. Moreover the ways of the birth and death of such kind of chaotic behavior in the dynamical systems have not been described except for the Shil'nikov–Turaev "blue-sky"catastrophe [Computers Math. Appl., 34, No 2-4, 1997, p.173]. In recent series of papers [Phys. Rev. Lett. 95, 2005, p. 144101; J. Exper. Theor. Phys. 102, 2006, p.355, etc] the methods to build up the systems demonstrating hyperbolic chaos were proposed.

Here we suggest and study several new bifurcation scenarios for one type of hyperbolic attractors – Smale–Williams solenoids. When changing the governing parameters in model systems the crisis or birth of attractor caused by different reasons can happen. The study of these bifurcations is the main topic of present work. The scenario of collision of a stable and an unstable hyperbolic sets, the birth of the hyperbolic solenoid by the transformation and destruction of the invariant circle, corruption of the hyperbolic strange set by arising singularities on it and other scenarios are described. These scenaria are carefully studied: a bifurcation analysis is carried out, the expansion of cycles embedded into the strange hyperbolic set is investigated, stable and unstable manifolds transformations are described, statistical regularities that accompany a construction or destruction of a hyperbolic set are calculated, scaling laws are found.

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The examples of interior conjugate diffeomorphisms that are nor neighborhood conjugate

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Suppose that transformations $f : M \to M$, $g : N \to N$ have invariant sets Λ_f, Λ_g, respectively.

Definition 1. The restrictions $f|_{\Lambda_f}$, $g|_{\Lambda_g}$ on Λ_f, Λ_g respectively are called *interior conjugate* if there is a homeomorphism $\varphi : \Lambda_f \to \Lambda_g$ such that $\varphi \circ f|_{\Lambda_f} = g \circ \varphi|_{\Lambda_f}$.

Definition 2. If the homeomorphism φ above can be extended to a homeomorphism $\varphi : M \to N$ or $\varphi : U(\Lambda_f) \to U(\Lambda_g)$ of some neighborhoods $U(\Lambda_f)$, $U(\Lambda_g)$ of the sets Λ_f, Λ_g, respectively, keeping the relation $\varphi \circ f|_{\Lambda_f} = g \circ \varphi|_{\Lambda_f}$, then $f|_{\Lambda_f}$ and $g|_{\Lambda_g}$ are called *neighborhood conjugate.*

Theorem. *There are compact 4-manifolds M, N and diffeomorphisms $f : M \to f(M) \subset M$, $g : N \to g(N) \subset N$ with one-dimensional expanding attractors Λ_f, Λ_g respectively such that the restrictions $f|_{\Lambda_f}$, $g|_{\Lambda_g}$ are interior conjugate but $f|_{\Lambda_f}$, $g|_{\Lambda_g}$ are not neighborhood conjugate.*

The construction of f is based on the Smale's construction of a diffeomorphism of $S^1 \times D^3$ into itself with a one-dimensional expanding attractor Λ_f that is locally homeomorphic to the product of the real line R and the standard Cantor set in в D^3. In order to construct g, we use the construction by Antoine. As a result, one gets a one-dimensional expanding attractor Λ_g that is locally homeomorphic to the product of the real line R and the Antoine necklace.

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Test of bifurcations for periodic solutions in the plane elliptic restricted three-body problem

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We examine a plane elliptic bounded three-body problem that is governed by the system of differential equations

$$\begin{cases} x'' - 2y' = \rho(x - \mu + \frac{\mu-1}{(x^2+y^2)^{3/2}}x - \frac{\mu}{((x-1)^2+y^2)^{3/2}}(x-1)), \\ y'' + 2x' = \rho(y + \frac{\mu-1}{(x^2+y^2)^{3/2}}y - \frac{\mu}{((x-1)^2+y^2)^{3/2}}y). \end{cases} \tag{1}$$

here $\rho = \frac{1}{1+\varepsilon\cos t}$, $\mu = \frac{m_2}{m_1+m_2}$, t is the true anomaly.

For the natural number $k \geq 2$ denote

$$\mu_k = \frac{1}{2} - \frac{\sqrt{27k^4 - 16k^2 + 16}}{6\sqrt{3}k^2}.$$

For $\varepsilon = 0$ and $\mu = \mu_k$ spectrum of the linearized problem at the triangular libration point $L_4(\frac{1}{2}, \frac{\sqrt{3}}{2})$ contains the number $\pm\frac{i}{k}$. This means that values $\varepsilon = 0$ and $\mu = \mu_k$ are points of bifurcation for the problem of searching for $2\pi k$ - periodic solutions of system (1) in a neighborhood of triangular libration point L_4. Here we provide new test to check such bifurcations as well as approximate formulas for arising periodic solutions.

Bifurcation in nonautonomous differential systems

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We consider some problems in the bifurcation theory of ordinary differential systems

$$x' = f(t; x; a),\ x \in \mathbb{R}^2$$

where a is a real parameter. The vector field f is a Birkhoff recurrent function of t, so in particular the time dependence might be almost periodic. We discuss an analogue of the Hopf bifurcation pattern in the context of such systems.

Stability of dynamical system under random perturbations

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Dynamical system under small stochastic perturbation is considered:

$$d\mathbf{y} = \mathbf{a}(\mathbf{y}, t)dt + \mu B(\mathbf{y}, t)d\mathbf{w}(t), \ t > 0; \quad \mathbf{y}|_{t=0} = \mathbf{x} \in \mathbb{R}^n.$$

Here $\mathbf{a}(\mathbf{y}, t)$ is a given vector–function in $\mathbb{R}^n$, $\mathbf{w}(t) \in \mathbb{R}^n$ is a n-dimensional Winer process, $B(\mathbf{y}, t)$ is a given matrix–function $n \times n$.

It is supposed that the point $\mathbf{y} = 0$ is a local asymptotic stable equilibrium of the unperturbed system $\dot{\mathbf{y}} = \mathbf{a}(\mathbf{y}, t)$, and the matrix of perturbation $B(0, t) \neq 0$. The problem of stability under small perturbation $|\mu| \ll 1$ is analyzed. The main result is as follows: The expectation of $|\mathbf{y}|^2$ along a trajectory $\mathbf{y} = \mathbf{y}_\mu(t, \mathbf{x})$ starting from point $\mathbf{x}$ near equilibrium remains small: $\mathbb{E}|(\mathbf{y}_\mu(t, \mathbf{x})|^2] < M(|\mathbf{x}|^2 + \mu^2)$ for a long time interval $0 < t < t_0\mu^{-2}$; $(M, t_0 = \text{const})$. Both the Lyapunov's functions and the method of parabolic equation are applied to prove the random stability. Stability of the auto-resonance phenomenon under random perturbations is discussed in the report.

Stability of autoresonance models

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Two model systems of the ordinary nonlinear nonautonomous differential equations are considered:

$$\frac{dr}{dt} = \sin\psi, \quad r\left[\frac{d\psi}{dt} - r^2 + \lambda t\right] = \cos\psi;$$

$$\frac{dr}{dt} = r\sin\psi, \quad \frac{d\psi}{dt} - r^2 + \lambda t = \cos\psi.$$

Here $\lambda = \text{const} > 0$. These equations describe an initial stage of capture into resonance for different nonlinear systems under weak excitation.

There exist two type of solutions: the first has bounded amplitude $r(t)$, the second has growing amplitude $r(t) \approx \sqrt{\lambda t}$ as $t \to \infty$. The solutions with growing amplitude correspond to auto-resonance phenomenon [1]. Stability of auto-resonance solutions is discussed in the report. Lyapunov functions are applied to prove the stability [2].

References

[1] Калякин Л.А. Асимптотический анализ моделей авторезонанса // Успехи мат. наук. 2008. 63, 5. С. 3-72.

[2] Султанов О.А. Устойчивость моделей авторезонанса при постоянно действующих возмущениях // Тр. ИММ УрО РАН, 2012. 18, 2. 254-264.

Three-color graph for topological classification of gradient-like diffeomorphisms on surfaces

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M. Peixoto [2] introduced "distinguished graph" to classify Morse-Smale flows on surfaces. A. A. Oshemkov and V. V. Sharko [1] suggested a new topological invariant for such flows — three-color graph — which is simpler for verifying an isomorphism than Peixoto's one.

Let G be a class of orientation preserving Morse-Smale diffeomorphisms on two-dimensional orientable surface M^2 satisfying to the following conditions:

1. for any distinct periodic points p, q of a diffeomorphism $f \in G$ the set $W^s_p \cap W^u_q$ is empty (i.e. f is gradient-like);

2. restriction of the diffeomorphism f to the unstable manifold $W^u(p)$ of any saddle periodic point p preserves orientation of $W^u(p)$.

We generalize the construction from [1] and associate with every diffeomorphism $f \in G$ a three-color graph $T(f)$. Any diffeomorphism $f \in G$ induces an automorphism S_f of the graph $T(f)$.

Theorem. *Two diffeomorphisms f, f' from class G are topologically conjugated iff exists isomorphism $\eta : T(f) \to T(f')$ such that $S'_{f'} = \eta S_f \eta^{-1}$.*

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References

[1] Oshemkov A. A., Sharko V. V. On the classification of Morse-Smale flows on two-dimensional manifolds // Matem. sbornik. V. 189. -No. 8. -1998.- p. 93-140.
[2] Peixoto M. M. On the classification of flows on 2-manifolds // Dynamical systems. New York: Academic Press, 1973.- P. 389-419.

Stability of auto-model cycles in a differential equation with large delay

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Consider a differential equation with large delay

$$\dot{u} = (1 + (-1 + ic)|u|^2)u + \gamma e^{i\varphi}(u(t - T) - u), \qquad T \gg 1. \qquad (1)$$

Here $u(t)$ is complex valued function, $\varphi \in [0, 2\pi)$, c is real parameter, γ and T are positive. This equation has a family of auto-model solutions of the cycle type. These solutions have asymptotics $u_{\rho,\omega} = (\rho + o(1)) \exp(i(\omega T + O(1))t)$ as $T \to \infty$. All points (ρ_*^2, ω_*) belong to a set in the nonnegative half-plane (ρ^2, ω) and obey the equation

$$(\rho^2 - 1 + \gamma \cos\varphi)^2 + (\omega - c\rho^2 + \gamma \sin\varphi)^2 = \gamma^2.$$

Denote this set as $L(c, \gamma, \varphi)$. Stability conditions for each auto-model solution of the cycle type are found. Geometry of sets corresponding to stable and unstable solutions in $L(c, \gamma, \varphi)$ is studied. It is proved that every $L(c, \gamma, \varphi)$ contains stable and unstable points. Moreover for $c = 0$ the stable region in $L(0, \gamma, \varphi)$ is simply connected for all $\gamma > 0$ and $\varphi \in [0, 2\pi)$.

Dynamics of a system with two large delays

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Consider the difference-differential equation with two delays

$$\dot{u} + u = au(t - T_1) + bu(t - T_2) + f(u, u(t - T_1), u(t - T_2)). \qquad (1)$$

Here $u(t) \in R$, a and b are real parameters, delays T_1 and T_2 are positive. The local (in a neighborhood of an equilibrium state) dynamics of system (1) is examined assuming both T_1 and T_2 to be sufficiently large and almost proportional, i.e.

$$T_1 = T \gg 1, \qquad T_2 = T(k + o(1)),$$

where k is positive constant. One of most interesting cases is $k = 1$, so delays T_1 and T_2 are close to each other.

All critical cases in problem of stability of equilibrium state have infinite dimension. In each of them by means of a special asymptotic method the problem (1) is reduced to a quasi-normal form that is to the problem that does not contain small or large parameters. The dynamics of quasi-normal form describes the behavior of solutions for the initial problem.

Quasi-normal form is a family of parabolic equations or a system of such equations with one or two spatial variables and periodic or anti-periodic boundary conditions. Exact type of quasi-normal form depends on signs of a, b and relation between small parameters.

In addition, the algebraic properties of k are very important. For irrational k quasi-normal form cannot be built. For rational $k = \frac{m}{n}$ quasi-normal form is family of equations of parabolic type with one or two spatial variables. Note that certain result depends of evenness of numerator m and denominator n.

Normal and quasi-normal forms for difference and difference-differential equations

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We study local (near zero equilibrium state) dynamics of difference and singular perturbed difference-differential systems. Critical cases in the stability problem of the equilibrium state have infinite dimension. Special nonlinear evolution equations are constructed, they serve as the normal forms. We show that their dynamics determines the behavior of solutions for the original system.

In the most interesting situations quasi-normal forms are families of boundary value problems depending on the “continual” parameters. Every

solution of a member of these families corresponds to the solution of the original system. We conclude that for the problems under consideration the phenomenon of hyper-multistability is typical. Hyper-multistability is unlimited increasing the number of steady states when small parameter tends to zero.

Similarities and differences in the dynamical properties of equations with discrete and continuous "time" and in discontinuous and smooth solutions of the original system of equations are revealed.

Shock-wave chaos

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We propose the following simple model equation that describes chaotic shock waves:

$$u_t + \frac{1}{2}(u^2 - u_s)_x = f(x, u_s)$$

It is given on the half-line $x < 0$ and the shock is located at $x = 0$ for any $t \geq 0$. Here $u_s(t)$ is the shock state and f is a given nonlocal source term. The equation is a modification of the Burgers equation that includes non-locality via the presence of the shock-state value of the solution in the equation itself. The model predicts steady-state solutions, their instability through a Hopf bifurcation, and a sequence of period-doubling bifurcations leading to chaos. This dynamics is similar to that observed in the one-dimensional reactive Euler equations that describe detonations. We present nonlinear numerical simulations as well as a complete linear stability theory for the equation.

Chaotic phenomena in the rubber rock'n'roller problem on a plane

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We consider the dynamics of the rubber rock'n'roller (a ball with a displaced gravity center) on a rough plane. The term "rubber" means that the vertical spinning of a body is impossible. The plane roughness means that a body moves without slipping. Rock'n'roller motions are described by the nonholonomic system being reversible with respect to several involutions, its number depends on the type of a mass center displacement. We demonstrate for this system the existence of complex dynamics which type depends on the kind of reversibility.

The dynamics of the rubber rock'n'roller is governed by the system of 6 equations which admits 3 integrals: energy, geometric and rubber, that reduces the problem dimension from 6 to 3. To visualize the dynamics of this system we construct two-dimensional Poincaré map in the Andoyer-Deprit variables.

If the ball mass center is displaced along all 3 body frame axis there is only one reversibility in the system connected with angular velocities inversion. In this case we found a strange attractor and (due to reversibility) strange repeller. Strange attractor has positive leading Lyapunov exponent and the sum of all exponents is negative.

If the ball mass center is displaced along 2 body frame axis, the system acquires an additional reversibility. Then the behavior of the system changes essentially. Here attractor and repeller are not separated. We suppose such chaotic behavior to be the mixed chaotic dynamics as in [1]. To justify this fact we find long periodic orbits of different types (stable and completely unstable) within this chaos.

References

[1] A. Delshams et al. Abundance of attracting, repelling and elliptic periodic orbits in two-dimensional reversible maps //Nonlinearity, **26** (2013), 1–33.

Hyperbolicity of minimizers for a random forced Hamilton-Jacobi equation

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We show that for a large class of randomly kicked Hamilton-Jacobi equations the unique global minimizer is almost surely hyperbolic. Furthermore, we prove that the unique forward and backward viscosity solutions, though in general only Lipshitz, are smooth in a neighborhood of the global minimizer. Related results in the one-dimensional case were obtained by E, Khanin, Mazel and Sinai (Annals, 2000). However the methods in the above paper are purely one-dimensional and cannot be extended to the case of higher dimensions. In this talk based on a joint paper with Ke Zhang we present a completely different approach.

Nonlinear dynamics of employment on the labor market

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In modern science, nonlinear mathematical models are used for study of socio-economic processes showing complex modes of dynamics. One of them is the employment of people in region.

For the study of problems ageism, discrimination and competition we developed basic model of different age professionals on the labor market. The model is a nonlinear system of ordinary differential equations describing the interaction between the different age groups of the employed population (ageism, discrimination, competition, assistance, cooperation) by type pairwise interactions in mathematical biology.

The simulation was shown complex modes of employment dynamics arise in crisis periods characterized by a negative migration balance, high death rate and increased competition between the different age groups in the regional labor market. Periodic fluctuations in the number of different age professionals can be observed in stable socio-economic development. In this case, fluctuations are mutually faded excluding the sharp spikes in the total number. One of the cause's periodic fluctuations in employment is the specific economic structure of the region. In case of development of innovative products trade will be more advantageous to hire fast educate young people. As a result may be formed employment discrimination of the older generation.

The proposed basic model of different age professionals reveals the nonlinear effects in the development of the labor market. In addition traditional economic analysis controlling the fluctuations and stabilization of the number of employed can be solved using the methods of applied nonlinear dynamics.

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A saddle in a corner - a model of atom-diatom chemical reactions

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A geometrical model which captures the main ingredients governing atom-diatom collinear chemical reactions is proposed. This model is neither near-integrable nor hyper-bolic, yet it is amenable to analysis using a combination of the recently developed tools for studying systems with steep potentials and the study of the phase space structure near a center-saddle equilibrium. The nontrivial dependence of the reaction rates on parameters, initial conditions and energy is thus qualitatively explained. Conditions under which the phase space transition state theory assumptions are satisfied and conditions under which these fail are derived. Extensions of these ideas to other impact-like systems and to other models of reactions will be discussed.

Shilnikov dynamics and partial differential equations

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Shilnikov dynamics in partial differential equations were likely first observed by Moore et al., Nature 303, 663-667 (1983), in their study of chaotic standing waves in doubly diffusive convection. In this talk I will describe the evidence for this conjecture and subsequent developments that that have largely confirmed this identification. I will also describe evidence for dynamics arising from homoclinic connections to strange invariant sets, also present in this system. Finally, I will comment on Shilnikov dynamics describing solitary waves in partial differential equations on the real line, and illustrate the results using several models of intracellular calcium transport of FitzHugh-Nagumo type, focusing on the mechanisms leading to the disappearance of such waves as a parameter varies.

On resonances and chaos in the system with a homoclinic ”figure-eight”

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We consider periodic in time perturbations of the nearly integrable asymmetric Duffing-van der Pol equation with a homoclinic ”figure-eight” of a saddle. For domains in the phase plane of the integrable system outside the ”figure-eight” we study the behavior of solutions which is closely connected with the problem of limit cycles for the autonomous equation and the thorough analysis in the resonance zones for the nonautonomous equation. The behavior of separatrices of the saddle fixed point of the Poincaré map in a small neighborhood of the unperturbed ”figure-eight” is established. Numerical results illustrate theoretical ones.

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Ring systems with hyperbolic attractors

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Three models of ring systems with an attractor of Smale-Williams type are proposed. The operation of these systems is based on phase manipulation. Equations governing the dynamics are chosen in such a way that the phase of the oscillations is doubled on each characteristic time period. This corresponds to the appearance of the Smale-Williams attractor in the phase space of the respective Poincaré map.

The first system is a non-autonomous ring composed of two linear oscillators and two nonlinear elements. The frequencies of the oscillators differ by factor of 2. One nonlinear element has a quadratic characteristic at low amplitudes and saturation at large amplitudes. The second one mixes the signal from the second oscillator with an external auxiliary signal, which is a sequence of radio-pulses with a smooth envelope.

The second scheme is a non-autonomous system composed of two oscillators and one nonlinear element. The frequency of the first oscillator is constant, but the frequency of the second oscillator is varied periodically; its minimum value is equal to the frequency of the first oscillator, and the maximum value exceeds it twice. The nonlinear element has a quadratic characteristic at low amplitudes and saturation at large amplitudes.

The third system is an autonomous ring scheme composed of a large number of van der Pol oscillators. Frequencies of the oscillators decrease along the chain smoothly, so that the frequencies of the first and the last oscillators differ by factor of 2. The oscillators are coupled in such a way that each element of the chain suppresses the oscillations of the others, and transfers the excitation to the next neighbor in the ring; specifically, the first and last oscillators are connected via a quadratic nonlinear element.

The proposed systems demonstrate phase doubling of oscillations on full cycle of signal transmission. Numerical results have been obtained that give evidence of existence of attractors of the Smale-Williams type in the phase space of the Poincaré maps for these systems.

The research is supported by RFBR grant No 12-02-31342.

Mathematical modeling of the processes of formation of nanostructures on the semiconductor surfaces under ion bombardment

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Sputtering process of amorphous or crystalline bodies (substrates) by low energy ion bombardment is one of the most effective methods for fabricating nanostructures on their surfaces. The key idea of this method is based on self-organization phenomena. As it was shown in periodic literature there are three types of surface morphology, which arise in experiments under different conditions. Normal incidence of ion beam leads to the formation of quantum dots or holes and oblique incidence causes the formation of aligned ripples on the surface. These structures have a number of useful properties and received increased attention in the recent years due to potential applications in nanotechnology. Thus the purpose of the present work is to describe the processes of pattern formation on the semiconductors surfaces under ion beam bombardment.

To reach this purpose the following problems were solved. We present the two dimensional nonlinear equation for describing the evolution of the surface height function with time. The exact solutions of this equation in one and two dimensional case were obtained. The «graphical classification» of these solutions in one dimensional case was performed. Based on the pseudospectral methods the numerical algorithms to solve one and two dimensional problems were developed. The numerical simulation of the pattern formation processes in one and two dimensions on the semiconductor surface was performed.

Generalized Burgers equation for description of nonlinear waves in liquid with gas bubbles

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Nonlinear waves in a liquid with gas bubbles are studied. Higher order terms with respect to the small parameter are taken into account in the derivation of the equation for nonlinear waves. A nonlinear differential equation is derived for long weakly nonlinear waves taking into consideration liquid viscosity, inter-phase heat transfer and surface tension. Additional conditions for the parameters of the equation are determined for integrability of the mathematical model. The transformation for linearization of the nonlinear equation is presented as well. Some exact solutions of the nonlinear equation are found for integrable and non-integrable cases. The normal form of the nonlinear equation is constructed with the help of the Kodama transformations. The nonlinear waves described by the nonlinear equation are numerically investigated.

Set of generators of quasi-periodic oscillations

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Quasi-periodicity is a form of behavior that can be observed in nonlinear dynamic systems from almost all areas of science and technology [1]. In the present paper one considers four different mechanisms of torus formation for a family of low-dimensional autonomous oscillators derived from the field of radio-engineering. The starting point for the present analysis is the three-dimensional oscillator system

$$\ddot{x} - (\lambda + z + x^2 - \beta x^4)\dot{x} + \omega_0^2 x = 0,\ \dot{z} = \mu - x^2$$

recently proposed by Kuznetsov et al. [2]. These equations represent an example of a minimal system that can exhibit quasi-periodic dynamics. One of the features of this system is the absence of equilibrium points. This fact shows the incompleteness of the suggested models. In the present paper we suggest two modifications of that system with one and two equilibrium points and describe features of the birth of a two-dimensional torus for each model. Particular emphasis is given to the birth of torus through a saddle-node bifurcation that eliminates the only stable cycle in the system and leaves the dynamics to settle down in the presence of a limit cycle of saddle type and a pair of equilibrium points of unstable focus/stable node and of stable focus/unstable node type. Also we suggest a new four-dimensional model with quasi-periodic behavior: for this system torus occurs as a result of Blue Sky bifurcation.

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References

[1] A. Pikovsky, M. Rosenblum, J. Kurths, *Synchronization. A Universal concept in Nonlinear Science* (Cambridge, 2001).

[2] A.P. Kuznetsov, S.P. Kuznetsov, and N.V. Stankevich, Commun. Nonl. Sci. Num. Sim., **15**, (2010) 1676-1681.

Invariant tori and complex dynamics in low-dimensional ensembles of oscillators

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The problem of the dynamics of coupled oscillators is fundamental. The case of two coupled oscillators is classical one [1]. But, already the case of three coupled oscillators is complicated [2,3]. In the present work we study ensembles of four or five elements. We discuss the following questions. How the results obtained for the phase model and for the original system correlate? What are the scenarios of birth of higher dimensional tori? What is the difference between the synchronization properties of the chain and network? To answer these questions, we discuss and compare the dynamics and bifurcations for the phase model and a system of van der Pol oscillators. Phase model permits to illustrate a saddle-node bifurcation of tori of increasing dimension. For the original system there is a new opportunity: quasi-periodic Hopf bifurcation [4]. The increase in the number of oscillators allows to observe the resonance Arnold web based on the tori of different dimension. We show that under certain conditions the network of five oscillators allows to realize Landau-Hopf scenario (the cascade of quasi-periodic Hopf bifurcations for the tori of higher and higher dimension). Comparison of the synchronization features for the chain and the network reveals a number of differences. The dynamics of networks is illustrated in the case of both in-phase and out-phase synchronization of the pairs of elements. That is of interest to the problems in laser physics [5].

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References

[1] Pikovsky A., Rosenblum M., Kurths J. Synchronization. Cambridge, 2001.
[2] Maistrenko Yu., et al. // Phys. Rev. Lett., 2004, vol. 93, 084102.
[3] Anishchenko V., et al. // Europhysics Letters, 2009, vol. 86, p.30003.
[4] Broer H., et al. // Regular and Chaotic Dynamics, 2011, vol. 16, p.154-184.
[5] Khibnik A.I., et al. // Physica D, 1998, vol. 111, p.295-310.

Synchronization and dynamics of three reactively coupled oscillators

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Synchronization is fundamental nonlinear phenomenon that is common both in science and engineering [1]. Dissipative coupling is the most examined case but other coupling are also possible, e.g. reactive coupling [2,3]. An interesting example of a physical system with reactive coupling was suggested in [4] as a chain of electromechanical nanoresonators. In [5] M.Cross & T.Lee suggested more realistic system with reactive coupling as chain of Penning traps. When chain is forced by noise, synchronization of clusters may spontaneously break and arise.

Reactive coupling is more sophisticated phenomenon than dissipative one [1]. The main feature of reactive coupling is multi-stability of coexisting syn-phase & asyn-phase modes. The case of two oscillators is well studied [2,3] but the case of three ones is not.

This paper considers three reactively coupled van der Pol oscillators and the related phase model. The space of oscillator frequencies has been scanned where areas of syn-phase, asyn-phase and mixed synchronization as well as two frequency and three frequency quasi-periodicity have been found via numerical simulations. The peculiar multi-stability of coexisting quasi-periodic and synchronous modes is distinctive feature of both models. Nonlocal bifurcation associated with alteration of cluster organization of chain are also discussed.

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References

[1] Блехман И.И. Синхронизация в природе и технике. М.: Наука, 1981.
[2] Ivanchenko M., et. al. // Physica D, 2004, vol. 189, № 1-2, pp. 8-30.
[3] Rand R., Holmes P.J. // Int. J. Non-Linear Mechanics, 1980, vol. 15, pp. 387-399.
[4] R. Lifshitz, et. al. // Phys. Rev. Lett., 2004, vol. 93, 224101 [4 pages].
[5] Lee T. E., Cross M. C. // Phys. Rev. Lett., 2011, vol. 106, 143001 [4 pages].

Hyperbolic chaos in physical systems

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Approaches are reviewed for constructing systems with hyperbolic chaotic attractors. In particular, we consider models driven with periodic pulses; dynamics consisted of periodically repeated stages, each corresponding to specific form of differential equations; design of systems of alternately excited oscillators transmitting excitation each other; the use of parametric excitation of oscillations; introduction of the delayed feedback. Examples of maps, differential equations, as well as simple mechanical and electronic systems are presented manifesting chaotic dynamics due to the occurrence of the uniformly hyperbolic attractors.

Additive noise does not destroy a pitchfork bifurcation

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It is well-known from [CF98: Crauel and Flandoli, "Additive noise destroys a pitchfork bifurcation Journal of Dynamics and Differential Equations 10 Nr. 2 (1998), 259-274] that adding noise to a system with a deterministic pitchfork bifurcation yields a unique random attracting fixed point with negative Lyapunov exponent for all parameters. Based on this observation, [CF98] concludes that the deterministic bifurcation is destroyed by the additive noise.

However, we show that there is qualitative change in the random dynamics at the bifurcation point. We associate this bifurcation with a breakdown of both uniform attraction and equivalence under uniformly continuous topological conjugacies, and with non-hyperbolicity of the dichotomy spectrum at the bifurcation point.

This is joint work with Mark Callaway, Doan Thai Son, and Martin Rasmussen (all at Imperial College London).

In memory of William P. Thurston, an evocation of his work on surfaces

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I would like to give a sketch of proof of Thurston's main theorem on homeomorphisms of surfaces whose statement is the following:

Let S be a closed orientable surface of genus $g \geq 2$. If φ is a homeomorphism of S, which is not isotopic to a periodic homeomorphism nor a homeomorphism keeping invariant an essential closed curve, then φ is isotopic to a pseudo-Anosov homeomorphism.

A remarkable feature of pseudo-Anosov homeomorphisms is that the number $N(T)$ of periodic points of period T grows exponentially with T, and still more remarkable is the fact that, for each period, they are minimizing the number of periodic points in their isotopy class. This last point follows from J. Nielsen fixed point theory.

For the proof of the above-mentioned statement, Thurston invented a compactification $\overline{\mathcal{T}}$ of the Teichmüller space of S on which the mapping class group acts naturally. This compact set is actually a $(6g-6)$-ball, where g is the genus of S. The points of the boundary are represented by measured foliations. According to the Brouwer fixed point theorem, $[\varphi]$, the isotopy class of φ, has at least one fixed point on $\overline{\mathcal{T}}$. The result depends on the position of that fixed point.

Reference

A. Fathi, F. Laudenbach, V, Poenaru, *Travaux de Thurston sur les surfaces,* Astérisque, vol. 66-67, Soc. Math. France, 1979 (2nd edition 1991); English translation by Djun Kim & Dan Margalit, Princeton University Press, 2012.

On some problems related to the Shilnikov's saddle-focus

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I intend to discuss several problems that are of interest for the theory of bifurcations for Hamiltonian, reversible and weakly dissipative systems. All this is related to the discovery of a complicated dynamics made in sixtieth by L.Shilnikov. Among these problems are the orbit structure near a homoclinic and heteroclinic connections for a reversible system, chaotic behavior on the singular level of Hamiltonian for system with a saddle-focus and existence of ellipticity, dissipative perturbations of an integrable Hamiltonian system with a homoclinic skirt of a saddle-focus.

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On topological classification of A-diffeomorphisms on M^3 with 2-dimensional surface basic sets.

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The talk is about the study of diffeomorphisms satisfying Axiom A by S. Smale on closed orientable connected 3-manifolds M^3 whose non-wandering set consists of two-dimensional hyperbolic surface basic sets. We show that for this class of diffeomorphisms the manifold M^3 is the locally trivial bundle over the circle with the fiber homeomorphic to 2-torus. Under certain conditions on the intersection of two-dimensional invariant manifolds of points from basic sets we obtain necessary and sufficient conditions of topological conjugacy of structurally stable diffeomorphisms from this class. Moreover, the problem of realization was solved for such diffeomorphisms.

Pursuit curves, the stationary Schrödinger's equation and forced vibrations

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It was recently noticed that the first two objects mentioned in the title are equivalent (when properly defined). I will describe this equivalence, as well as an interesting connection between pursuit curves and high frequency vibrations.

Various strategies of coupling and undersampling of chaotic sequences to improve their unpredictability

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Nowadays there exists an increasing demand for new and more efficient pseudo random number generators (PRNG). These demands arise from different applications, such as multi-agents competition, global optimization via evolutionary algorithms, secure information transmission, [1, 2].

During the last decade, it has been emphasized that the undersampling of sequence of chaotic numbers [3] is an efficient tool in order to build pseudo-random number generators (PRNG). Randomness appears to be emergent property of complex systems of coupled chaotic maps when an undersampling process is added to the output of the corresponding dynamical system [4]. Several kinds of coupling and undersubsampling can be considered as for example ultra-weak coupling or ring coupling; chaotic or geometric undersampling. An ultra-weak coupling recovers chaotic properties of 1-dimensional maps when computed with floating numbers or double precision numbers. Chaotic under-sampling with thresholds based on one component of the coupled system adds random properties to the chaotic sequences. Double threshold undersampled sequence (i.e., using both thresholds of different nature) improves such random properties. Ring coupling deals when p 1-dimensional maps are constrained on a torus [5], this coupling can directly generate random numbers, without sampling or mixing, provided the number p of maps is large enough, although it is possible to combine these processes with it. However in lower dimension 2 and 3, the chaotic numbers are not equidistributed on the torus. Therefore an original geometrical undersampling based on the property of piecewise linearity of the invariant measure of system is needed to recover the equidistribution. This geometric undersampling is very effective for generating parallel streams of pseudo-random numbers with a very compact mapping [6].

We explore these various strategies of both coupling and undersampling of chaotic sequences in the scope of Chaotic multi stream PRNG, eventually we improve their results combining several CmsPRNG.

References

[1] **S. Hénaff, I. Taralova, I., & R. Lozi,** "Exact and Asymptotic Synchronization of a new weakly coupled maps system", *Journal of Nonlinear Systems and Applications*, 1, 87-95, (2010).

[2] **R. Lozi, S. El Assad and I. Taralova**, "Chaotic Generator Synthesis: Dynamical and Statistical Analysis", In *Proceeding of The 7th International Conference on Internet Technology and Secured Transactions (ICITST),* 10-12 December 2012, London, GB, (Workshop ISTP-11, 2nd International Workshop on Information Security and Practice), pp. 56-59.

[3] **R. Lozi,** "New Enhanced Chaotic Number Generators", *Indian Journal of Industrial and Applied Mathematics*, vol.1, No.1, pp.1-23, 2008.

[4] **R. Lozi**, "Emergence of randomness from chaos", *International Journal of Bifurcation and Chaos*, Vo. 22, No. 2 (2012) 1250021-1/1250021-15.

[5] **I. Taralova, A. Espinel, & R. Lozi**, "Dynamical and statistical analysis of a new Lozi function for random numbers generation", in *Proceeding of Physcon 2011*, León, Spain, 5-8 september, IPACS open Access Electronic Library.

[6] **I. Taralova, & R. Lozi**, "From chaos to randomness via geometric undersampling", In *Proceeding of European Conference on Iteration Theory* (ECIT 2012), 9-15 September 2012, Ponta Delgada, Açores, Portugal p. 44.

Chaos generated by a dynamic system alike Duffing equation

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This paper considers a second order nonlinear differential equation

$$x'' + p(x) = \gamma \cos \epsilon t \tag{1}$$

where $x' = dx/dt$, $p(x)$ is an $S-$shaped polynomial, and γ and ϵ are positive constants. One of such equations is the well known Duffing equation without damping in which $p(x) = x^3 - x$ or a third order polynomial. Many researchers have investigated the rich phenomenon of its solutions numerically. There are also some analytical results based on the perturbation method and the Poincare maps. However, there are still more questions to be investigated. For example, if the solutions are chaotic, can we find the pattern of such solutions? This paper uses a piecewise linear function to approximate the polynomial $p(x)$ to explore certain types of chaotic solutions of equation (1). As an example, we study the equation

$$x'' - f(x) = \gamma \cos \epsilon t \tag{2}$$

where $f(x)$ is a piecewise linear function

$$f(x) = \begin{cases} x \text{ for } |x| \leq 1 \\ 2 - x \text{ if } x > 1 \\ -2 - x \text{ if } x < -1 \end{cases} \tag{3}$$

By the Weierstrass approximation theorem, for any $\Delta > 0$, there exists a polynomial $p(x)$ such that $|p(x) - f(x)| < \Delta$ for all x in a finite interval $[\alpha, \beta]$. This also means that $f(x)$ can be used to approximate the polynomial $p(x)$ on any finite interval.

Note that the piecewise linear function $f(x)$ is Lipschitz conditioned and therefore, the existence and uniqueness theorem of solutions to (2) can be applied. In addition, solutions of (2) continuously depend on its initial values, parameters, and the function $f(x)$ on any finite interval of t.

Precisely, we can take Δ small enough so that the solutions of equations (2) and (1) and their first order derivatives can be sufficiently close over any fixed finite interval. In this way, the behavior of solutions of (1) is determined by the solutions of (2) on the finite interval. The analysis in this paper shows that the behavior of the solutions of (2) on the finite interval will determine the behavior of the solutions for $-\infty < t < \infty$, which becomes chaotic in the sense that there exist a countable set of periodic solutions and a non-countable set of bounded non-periodic solutions. In this paper, the N-shaped function $(-f(x))$ is used to approximate the S-shaped polynomial. We will then observe the coexistence of the periodic and non-periodic solutions of a generalized Duffing equation. The coexistence of periodic and non-periodic solutions has been introduced by Shilnikov in some three dimensional dynamical systems. This paper shows that the similar chaotic behavior may occur in a two dimensional dynamical system.

Analytical Solution of Periodic Motions to Chaos in Nonlinear Dynamical Systems

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Analytical solutions of period-m flows and chaos in nonlinear dynamical systems are presented through the generalized harmonic balance method. The mechanism for a period-m flows jumping to another period-n motion in numerical computation is found. The period-doubling bifurcation via Poincare mappings of dynamical systems is one of Hopf bifurcations of periodic flows. The stable and unstable period-m motions can be obtained analytically. In addition, the stable and unstable chaos can be achieved analytically. The methodology presented in this paper is independent of small parameters. The nonlinear damping, periodically forced, Duffing oscillator was investigated as an example to demonstrate the analytical solutions of periodic motions and chaos.

Chimera states for repulsively coupled phase oscillators

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Chimera states represent remarkable spatio-temporal patterns of phase-locked oscillators coexisting with irregular chaotically drifting ones. Surprisingly, they typically develop in arrays of coupled identical oscillators without any sign of asymmetry - as a manifestation of internal nonlinear nature of dynamical networks.

We discuss the appearance of the chimera states for repulsively coupled phase oscillators of Kuramoto-Sakaguchi type, i.e., when the phase lag parameter $\alpha > \pi/2$ and hence, the network coupling works against synchronization. We find that chimeras exist in a wide domain of the parameter space as a cascade of the states with increasing number of regions of irregularity - so-called chimera's heads.

We also study numerically the origin of the chimera states and show that they grow from so-called multi-twisted states. Three typical scenarios for the chimera birth are reported: 1) via saddle-node bifurcation on an invariant curve, also known as SNIC or SNIPER, 2) via blue-sky catastrophe when two periodic orbits, stable and saddle, approach each other and annihilate eventually in a saddle-node bifurcation, and 3) via homoclinic transition, when the unstable manifold of a saddle comes back crossing the stable manifold giving rise to Shilnikov homoclinic chaos.

Bifurcations and universality for rotation sets of Lorenz type systems

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We consider symbolic models for one-dimensional maps of Lorenz type with positive topological entropy in the form of countable topological Markov chains (Hofbauer models) and study behavior of rotation sets in one-parameter families of perturbed (in general, multidimensional) maps of Lorenz type. More precisely, let

$$\Phi_\lambda(y_n, y_{n+1}, \ldots, y_{n+m}) = 0,\ n \in \mathbf{Z},$$

be a difference equation of order m with parameter λ. It is asuumed that the non-perturbed operator Φ_{λ_0} depends on two variables, i.e., $\Phi_{\lambda_0}(y_0, \ldots, y_m) = \psi(y_N, y_M)$, where $0 \leq N, M \leq m$ and ψ is a piecewise monotone piecewise C^2-function. It is also assumed that for the equation $\psi(x, y) = 0$, there is a branch $y = \varphi(x)$ which represent a one-dimensional Lorenz-type map. We prove approximation results for the problem on continuous dependence of the rotation set under multidimensional perturbations. Numerical results show universality phenomena in bifurcations responsible for birth of nontrivial rotation intervals with respect to renormalized subsystems in one-parameter families of Lorenz-type maps. Our technique is based on approximations of topological entropy and maximal measures represented by countable topological Markov chains [1] and also, on continuation of chaotic orbits for perturbations of singular difference equations [2].

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References

[1] Malkin M.I. *Countable Topological Markov Chains with Meromorphic Zeta-Functions.* Random & Computational Dynamics 2 (1994), no. 3-4, 247-259.

[2] Juang J., Li M.-C., Malkin M.I. *Chaotic difference equations in two variables and their multidimensional perturbations.* Nonlinearity, 21 (2008), 1019-1040.

The structure of dendrites admitting the existence of an arc horseshoe

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Let X be a dendrite or a graph, $f : X \to X$ is a continuous map. We say that f *has*

• *a horseshoe*, if there exist nonempty disjoint sub-continua $A, B \subset X$ such that $f^n(A) \cap f^n(B) \supset A \cup B$;

• *an arc horseshoe*, if there are arcs $A, B, C \subset X$ such that $A, B \subset C$ and A, B form a horseshoe for f.

In [1] one demonstrates that for a continuous map f on a graph the positivity of a topological entropy of f is equivalent to the existence of an arc horseshoe for some iteration of f. In [2,3] one constructs examples of continuous maps on dendrites with a positive topological entropy that have a horseshoe, but no iteration of the map has an arc horseshoe. We say that *a dendrite X admits the existence of an arc horseshoe*, if for any continuous map $f : X \to X$ having a horseshoe that exists a natural number $n \geq 1$ such that f^n has an arc horseshoe. In the report the structure of dendrites admitting the existence of an arc horseshoe is investigated. We hence deduce conditions for the existence of homoclinic points for a continuous map of dendrites, characteristic for a continuous map of a segment.

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References

[1] J. Libre, M. Misiurewicz. Horseshoes, entropy and periods for graph maps. *Topology*, **32** (2003), 649–664.

[2] E.N. Makhrova. Homoclinic points and topological entropy of a continuous mappings of a dendrite. *Journal of Math. Scieces*, **158** (2009), 241–248.

[3] Zd. Kocan, V. Korneka-Kurkova, M. Malek. Entropy, horseshoes and homoclinic trajectories on trees, graphs and dendrites. *Ergodic theory&Dyn. Sys.*, **XX**(2010), 1–11.

Exponential speed of mixing for skew-products with singularities

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Let $f : [0;1] \setminus \{1/2\} \times [0,1]; \to [0;1] \times [0;1]$ be the C^∞ endomorphism given by

$$f(x;y) = \left(2x - [2x], y + \frac{c}{x - 1/2} - [y + \frac{c}{x - 1/2}]\right),\ c \in \mathbb{R}^+.$$

We prove that f is topologically mixing and if $c > 1/4$ then f is mixing with respect to Lebesgue measure. Furthermore the speed of mixing is exponential. This skew-product can be seen as a toy model for flows with singularities attached to regular orbits. In particular, it is a toy model for a Lorenz-like flow on 3-manifolds.

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On the existence of homoclinic orbits at periodic reversible Hamiltonian Hopf bifurcation

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We study a generic one parameter Hamiltonian unfolding of a 2π-periodic smooth nonautonomous 4-dimensional Hamiltonian system that has a 2π-periodic solution. We suppose at the critical value of parameter $\varepsilon = 0$ the system linearized at periodic orbit to have a pair of double multipliers on the unit circle. Assuming the absence of strong resonances the Hamiltonian is normalized up to the terms of the 4th order, denote it as H_4. The latter Hamiltonian is autonomous integrable, it has the same form as for the Hamiltonian Hopf bifurcation near an equilibrium. After rescaling variables and parameter H_4 has the form

$$\frac{\delta}{2}(p_1^2 + p_2^2) + \omega(p_1q_2 - p_2q_1) + \frac{\varepsilon}{2}(q_1^2 + q_2^2) +$$

$$+(q_1^2 + q_2^2)\left[A(q_1^2 + q_2^2) + B(p_1q_2 - p_2q_1) + C(p_1^2 + p_2^2)\right],$$

where δ, A, B, C depend on ε, $\delta(0)$ and $A(0)$ do not vanish.

We consider the system when $\varepsilon < 0$, then the fixed point of corresponding Poincaré mapping is of a saddle-focus type (its eigenvalues are quadruple of complex numbers out of the unit circle). Poincaré mapping for the system is a perturbation of that obtained by 2π-shift along orbits of the Hamiltonian system X_{H_4} with Hamitonian H_4. When $A > 0$ stable and unstable 2-dim manifolds of the saddle-focus for X_{H_4} coalesce forming a set topologically equivalent to a sphere with two points identified. The same is true for the related Poincaré mapping. This set is filled with homoclinic orbits to the fixed point. Stable and unstable manifolds of saddle-focus for the Poincaré mapping of the full system are generally split.

Our main result is the following theorem.

Theorem 1. *Let the system in addition be reversible with respect to a smooth involution interchanging stable and unstable manifolds of the saddle-focus. If $A > 0$, then stable and unstable manifolds of saddle-focus for the Poincaré mapping of full system intersect along at least two symmetric homoclinic orbits.*

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Scaling properties and limit behaviour of the largest islands the Chirikov's Standard Map far away from Greene's limit

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The well-known Chirikov's Standard Map

$$T : \begin{pmatrix} x \\ y \end{pmatrix} \mapsto \begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix} = \begin{pmatrix} x + \bar{y} \\ y + k\sin(2\pi x) \end{pmatrix}$$

has rotational invariant curves (homotopically nontrivial) for $2\pi k < 2\pi k_G \approx 0.971635...$, Greene's threshold. For values $k > k_G$ chaotic orbits are not longer bounded in the momentum y and a chaotic sea seems to fill up the phase space as k increases.

Near integer and half-integer values of k there are stable accelerator mode orbits which seem to have the same structure after a suitable scaling. These are the largest stable objects in the phase space.

The scaling of such stable orbits is obtained. This allows us to derive 'limit' maps for both integer and half-integer values of the parameter k. Numerical estimates on the measure of the set of points in these stable islands as k varies are also given including, in particular, the measure of the regular zone of the former 'limit' maps, depending on a suitably scaled parameter.

Sequential switching activity in ensembles of inhibitory coupled oscillators

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The heteroclinic cycles and channels are mathematical images of sequential switching activity in neural ensembles. We present a phenomenological model of such activity. The model is based on coupled Poincaré maps. The existence of heteroclinic cycles and channels is shown.

Many dynamic processes in ensembles of coupled oscillatory systems demonstrate sequential switching activity between the individual elements and (or) groups of elements. Such sequential activity in neural network may be associated with different physiological functions of the nervous system [Komarov M.A., Osipov G.V., Suykens J.A.K. and Rabinovich M.I., CHAOS, 19, 2009, 015107].

Sequential activity in oscillatory networks can be considered in terms of nonlinear dynamics. Thus, formation of a stable heteroclinic cycle in the phase space of corresponding dynamical system. This dynamical system simulates the activity of the network, can be the cause of such activity [Ashwin P., Burylko O. and Maistrenko Y., Physica D, 237, 2008, 454]. The basis principle of the generation of sequential switchings activity is the winnerless competition principle [Seliger P., Tsimring L.S. and Rabinovich M.I., Phys. Rev. E, 67, 2003, 011905]. The essence of this principle is the existence in the phase space of stable heteroclinic cycle between the trajectories of saddle type (saddle equilibrium points, saddle limit cycles) [Afraimovich V.S., Rabinovich M.I. and Varona P., Int. J. Bif. and Chaos, 14, 2004, 1195]. A passage of the phase point in a neighborhood of a certain saddle trajectory corresponds to activation of specific oscillators or groups of them. All such trajectories in the vicinity of heteroclinic cycle form heteroclinic channel. Thus, a stable heteroclinic channel in the phase space can be considered as a mathematical image of the sequential switching activity in ensembles of oscillators.

We study the question of the existence of a stable heteroclinic cycle between *saddle limit cycles.*

On energy function for structurally stable diffeomorphisms on surfaces

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Energy function for dynamical system is a smooth function which decreases along trajectories out of the chain recurrent set, it is constant on the chain components and the set of its critical points coincides with the chain recurrent set. The first construction of an energy function was given by S. Smale [3] who proved in 1961 the existence of a Morse energy function for gradient-like flows on a closed n-manifold ($n \geq 1$). In 1977 D. Pixton [2] constructed a Morse energy function for any Morse-Smale diffeomorphisms on surfaces. He also noticed in [2] that such function does not exist in general even for Morse-Smale diffeomorphisms on closed 3-manifold. In series of works (see for example [1]) necessary and sufficient conditions for existence of energy function for any Morse-Smale diffeomorphisms on 3-manifolds were found.

We consider a class $S(M^2)$ of orientation preserving structurally stable diffeomorphisms of a closed orientable surface M^2 and assume their nonwandering set to admit non-trivial one-dimensional attractors and repellers. The following theorem holds.

Theorem *Any diffeomorphism $f \in S(M^2)$ possesses a smooth energy function which is Morse function out a neighborhood of union of nontrivial basic sets.*

The results were obtained in collaboration with V.Z. Grines and O.V. Pochinka. The work was supported by the Government of Russian Federation (grant no. 11. G34.31.0039), by the Russian Foundation for Basic Research (grants no. 11-01-12056-ofi-m-2011 and no. 12- 01-00672-a), and by the Ministry of Education and Science of Russian Federation (state contract no. 1.1907.2011 for 2012-2014).

References

[1] В. З. Гринес, Ф. Лауденбах, О. В. Починка, Динамически упорядоченная энергетическая функция для диффеоморфизмов Морса–Смейла на 3-многообразиях // Тр. МИАН. — 2012. – V. 278. – P. 34-48

[2] Pixton D. Wild unstable manifolds // Topology. — 1977. – V.16(2). –

P.167-172.
[3] Smale S. On gradient dynamical systems // Ann. Math. – 1961. – P.199-206.

On bifurcations in resonance zones

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Bifurcations in resonant zones for Hamiltonian systems with 3/2 degrees of freedom being close to nonlinear integrable and symplectic maps of the cylinder are discussed. The resonances are divided into non-degenerate and degenerate ones [1]. The most interesting bifurcations are related with degenerate resonances. For symplectic non-twist maps of the cylinder, two bifurcation scenarios are described: the formation of "loops" and of "vortex pairs", these scenarios were discovered first in [2] (see, also [3]). Numerical computations for Hamiltonian systems also show the presence of the "vortex pairs". The second approximation of the averaging method for the system is considered. If perturbations are not small both scenarios are realized.

For quasi-Hamiltonian systems similar bifurcations in resonant zones are also discussed. It is worth noting that the problem on degenerate resonances was considered for the first time in [1].

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References

[1] A.D. Morozov & L.P. Shil'nikov. On nonconservative periodic systems similar to two-dimensional Hamiltonian ones// Prikl. Mat. i Mekh (in Russian), 1983, **47**, No. 3, 385-394.
[2] J. E. Howard and S. M. Hohs. Stochasticity and reconnection in Hamiltonian systems //Phys. Rev. A29 (1984), 418.
[3] James E. Howard and Albert D. Morozov. A Simple Reconnecting Map// Regular and Chaotic Dynamics, 2012. Vol. 17, № 5, pp. 418-431.

Asymptotic comparison principle and its application for reaction-diffusion-advection equations with fronts

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We consider some cases of initial boundary value problem for the equation

$$\varepsilon^2 \Delta u - \frac{\partial u}{\partial t} = f(u, \nabla u, x, \varepsilon), \quad x \in \mathcal{D} \subset R^N,\ t > 0, \tag{1}$$

which plays important role in many applications and is called reaction-diffusion-advection equation. It is known that the dynamics of processes described by this equation is significantly determined by the stationary states of the problems. We present our new approach to h boundary and internal layers. Particularly the cases when equation (1) is semilinear or quasylinear are considered. Among others we discuss the following problems:

1. Existence and Lyapunov stability of stationary solutions.

2. The problem of stabilization of the solution of initial boundary value problem.

Our investigations are based on asymptotic method of differential inequalities and recent development of general scheme of this method will be presented. This scheme uses so-called positivity property of the operators producing formal asymptotics.

On the base of our approach we present some recent results on stabilization of the solutions classes for the problems, which are based on our results for moving front type solutions:

1. Solutions with boundary and internal layers

$$\varepsilon^2 \Delta u - \frac{\partial u}{\partial t} = f(u, x, \varepsilon), \quad x \in \mathcal{D} \subset R^N,\ t > 0,$$

$$u = h(x), \quad x \in \partial\mathcal{D},\ t > 0,$$

2. Solutions with internal layers

$$\varepsilon^2 \Delta u - \vec{A}(u, x) \nabla u - \frac{\partial u}{\partial t} = f(u, x, \varepsilon), \quad x \in \mathcal{D},\ t > 0,$$

$$u = h_i(x), \quad x \in \partial\mathcal{D}_i,\ i = 1, 2,\ t > 0.$$

The results of the work is a further development of the results of papers [1]-[3].

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References

[1] Nefedov N.N., The Method of Differential Inequalities for Some Classes of Nonlinear Singularly Perturbed Problems with Internal Layers, *Differ. Uravn.*, 1995. vol. 31. no. 7. pp. 1142-1149.
[2] A.B. Vasileva, V.F. Butuzov, N.N. Nefedov. Singularly Perturbed problems with Boundary and Internal Layers. Proceedings of the Steklov Institute of Mathematics, 2010, Vol.268, pp.258-273.
[3] Nefedov N.N., Nikitin A.G., Petrova M.A., Recke L. Moving fronts in integro-parabolic reaction-diffusion-advection equations *Differ. Uravn.*, 2011. vol. 47. no. 9. pp. 1-15 (Differentialnie Uravn. v.47, No. 9, pp. 1305-1319 (in Russian)).

On mechanisms of destruction of adiabatic invariance in slow-fast Hamiltonian systems

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Adiabatic invariants are approximate first integrals of systems with slow and fast motions (slow-fast systems). In the talk it is planned to present a review of currently known mechanisms of destruction of adiabatic invariance. It is planned to consider destruction of adiabatic invariance due to captures into resonances, repulsion from resonances and scattering on resonances, passages through a separatrix, and changes of modes of motion in systems with elastic collisions. It is planned to consider examples of manifestation of these mechanisms in problems related to charged particles dynamics.

Heteroclinic contours and self-replicated wave patterns in neural networks with complex-threshold excitation

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We present some results on study of spatio-temporal dynamics of systems mimicking collective behavior of one and two dimensional ensembles of electrically coupled neurons with nonlinear recovery properties. We show that the 1-D neural lattice is capable of displaying self-sustained spiking patterns emerging from an interplay between unstable nonlinear waves (wave fronts, pulses and solitary bound states). The local oscillations in such regime represent chaotic sequences of pulses which, however, are organized in a fractal triangular-like space-time pattern. On the other hand, if the waves is stable and display particle-like behavior in collisions it may lead to the appearance of complex self-replicated spatio-temporal patterns. The study of 2-D system showed that it supports formation of two distinct kinds of stable two dimensional spatially localized moving structures without any external stabilizing actions - regular and polymorphic structures. The regular structures preserve their shape and velocity under propagation while the shape and velocity as well as other integral characteristics of polymorphic structures show complex enough temporal behaviour. The correspondence between the structures and trajectories in multidimensional phase space associated with the system is given. Bifurcation mechanisms leading to loss of stability of regular structures as well as to transition from one type of polymorphic structures to another are indicated. The mechanisms of formation of regular and polymorphic structures is discussed.

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Ring system with the complex analytical dynamics phenomena

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A model system manifesting phenomena peculiar to complex analytical maps (such as Mandelbrot and Julia sets) is proposed. The system is a non-autonomous ring cavity with nonlinear elements and filters. The external pumps with frequency ω and complex slow amplitudes C and D excites the signal in the ring. Nonlinear elements are segments of medium of length R_1 and R_2. In these media due to the pump with frequency ω the parametric excitation of the component with frequency 2ω and interaction of two these components takes place according to the model:

$$i\dot{a}(x) = \alpha a^* b, \quad i\dot{b}(x) = \beta a^2, \tag{1}$$

where $a(x)$ and $b(x)$ are complex slow amplitudes of the components with frequency ω and 2ω, x is a coordinate along a segment of medium, α and β are parameters. Model contains two nonlinear elements, combined with two filters (with frequency ω and 2ω). This construction is organized in such a way, that at each loop of traveling of a signal in the ring the following mechanism realizes: 1) on the first nonlinear element the amplitude a of the signal with frequency ω transmits by quadratic term in (1) as primary amplitude b of the component with frequency 2ω and $b(R_1) \sim a^2(0) = Z^2$; 2) the first filter cuts off the component a, which is replaced then by external pump D; 3) the signal with the components with amplitude $a(0) = D$ and $b(0) = b(R_1)$ comes to the second nonlinear element, in which the backward transmission of the primary amplitude from b to a occurs and $a(R_2) \sim D^* b(R_1) \sim D^* Z^2$; 4) second filter cuts off the component b with frequency 2ω and external pump C accrues to component a. Thus the behavior of the slow amplitude of signal in the stroboscopic section corresponds to complex map $Z' = C + D^* Z^2$.

The work was supported by RFBR Grant No.12-02-31342 (M.A.O., O.B.I.) and RFBR Grant No.12-02-00342 (S.P.K.).

Bifurcations and attractors in six-dimensional system of two dynamically coupled van der Pol-Duffing oscillators

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The study of routes in which a system undergoes a transition to complicated behavior is one of important problem of nonlinear dynamics. Special attention in this field is paid to chaotic oscillations that are sensitive to the system initial conditions and system parameters. In particular, much work had been devoted to the birth of chaotic behavior via bifurcations of homoclinic orbits, limit cycles and low- or high-dimensional quasi- periodic attractors. Significant progress in understanding the bifurcation features for various scenarios had been achieved through the works of D. Ruelle, F. Takens, V.S. Afraimovich, S.V. Gonchenko, D.V. Turaev, L.P. Shil'nikov et al. We investigate the bifurcation mechanisms leading to emergence of quasi-periodic and chaotic attractors in phase space of two Van der Pol-Duffing oscillators inertially coupled through common linear system. In mechanics, this type of indirect coupling comes from Huygens problem related to dynamics of two pendulum clocks attached to a common support beam. It should be noted that besides the originally observed regime where pendula swung in anti-phase only, recently, the richest dynamics have been revealed. We present some mechanisms leading to this variety of complicated behavior, and demonstrate that besides the periodic sets, in the phase space of the system the existence of various quasi-periodic and chaotic attractors is observed. To study the properties of the attractors and mechanisms of their destruction we numerically calculate the Lyapunov spectrum and analyze the portraits in Poincaré sections. We discuss the routes leading to chaos and estimate the dimension of chaotic attractors observed for various values of system parameters. The existence of high-dimensional chaotic attractors is shown. The role of Duffing nonlinearity in appearance of complicated behavior is examined in detail. We obtain the conditions of the total escape of the trajectories to infinity.

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1:3 resonance of multi-site breathers in Klein-Gordon lattices

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I will review stability and resonances of multi-site breathers in the discrete Klein-Gordon equation with a small coupling constant. We show that the stability of multi-site breathers depends on the phase difference between the oscillations and change between soft and hard nonlinear potentials. We also study a symmetry-breaking (pitchfork) bifurcation of multi-site breathers in soft quartic potentials near the points of 1:3 resonance.

Complex collective dynamics in oscillator populations

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Synchronization in populations of coupled oscillators is a phenomenon relevant for many fields of science. In some cases the dynamics of the ensemble can be described by a finite number of global variables, so that a dynamical system with a few degrees of freedom has to be analized. I give examples of such a reduction and of complex behavior of global variables, including heteroclinic cycles, multistability, strange attractors.

Nonhyperbolic shadowing

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We apply the method of Lyapunov functions to obtain sufficient conditions of shadowing near nonhyperbolic fixed points. In some cases, the method allows us to give exact estimates of distance between a pseudotrajectory and the shadowing trajectory.

We also study the shadowing property near nonisolated fixed points; in this case, one has to control the “one-step"errors of pseudotrajectories.

On simple isotopic classes of Morse-Smale diffeos on 3-sphere

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We deal with the Palis-Pugh problem [1] on the existence in the space of diffeomorphisms on smooth closed manifold M^n of an arc with finite or countable set of bifurcation points joining two given Morse-Smale diffeos. Newhouse and Peixoto [2] have shown that for flows such arc exists for any n, moreover the arc is simple. *Simple* means that the arc contains at most finitely many points outside of Morse-Smale flows and, roughly speaking, for these points the related diffeomophism deviates in the least possible way from the structurally stable one. However there are isotopic diffeomorphisms which can not be joined by a simple arc. One says that two Morse-Smale diffeomorphisms are in the same *simple isotopy class*, if they can be connected by a simple arc. In [3] it was established that any “sink-source” diffeomorphisms belong to the same simple isotopic class. Here we find necessary and sufficient conditions for a Morse-Smale diffeomorphism without heteroclinic intersection on 3-sphere to be joined by a simple arc with “sink-source” diffeomorphism. The results were obtained in collaboration with V. Grines.

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References

[1] Palis J., Pugh C. Fifty problems in dynamical systems // Lecture Notes in Math. – 1975. – V. 468. – P. 345–353.
[2] S. Newhouse and M. M. Peixoto, There is a simple arc joining any two Morse-Smale flows // Asterisque, 31. – 1976. – P. 15–41.
[3] Bonatti Ch., Grines V., Medvedev V., Pochinka O. Bifurcation of Morse-Smale diffeomorphisms with wild embedded separatrices // Proceedings of MIAN. – 2007. – V. 256. – P. 54–69.

The rate of convergence in Birkhoff theorem for non-uniformly hyperbolic dynamical systems

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In this report we deal with dynamical systems admitting Markov tower extensions with exponential return times in the sense of [1]. These systems have a mixing (SRB) measure in case of an additional aperiodicity assumption on the return time.

Let $(\mathcal{M}, \mu; T)$ be a mixing dynamical system of the described type, $f : \mathcal{M} \mapsto \mathbb{R}$ be Hölder continuous function with the mean value $\mu(f) = \int_{\mathcal{M}} f \, d\mu$. For ergodic averages

$$A_n f(x) = \frac{1}{n} \sum_{k=0}^{n} f(T^k x), n \in \mathbb{N}, x \in \mathcal{M}$$

there is established in [2] the following large deviations result. There exists $\theta_0 > 0$ such that function $e(\theta) := \lim_{n\to\infty} \frac{1}{n} \ln \mu(e^{n\theta A_n f})$ is real analytic in the interval $(-\theta_0, \theta_0)$, $e'(0) = \mu(f)$ and for any small enough $\varepsilon > 0$

$$\lim_{n\to\infty} \frac{1}{n} \ln \mu \{x \in \mathcal{M} : |A_n f(x) - \mu(f)| \geq \varepsilon\} = -\inf_{t \in R_\varepsilon} I(t), \qquad (1)$$

where $R_\varepsilon = (e'(-\theta_0), e'(0) - \varepsilon] \cup [e'(0) + \varepsilon, e'(\theta_0))$ and $I(t) = \sup_{\theta \in [-\theta_0, \theta_0]} \theta t - e(\theta)$ is the Legendre transform of $e(\theta)$.

Our main result based on the continuity of $I(t)$ and the technics arising from [3] is that the limit (1) holds also for the quantity $\mu \left\{ x \in \mathcal{M} : \sup_{k \geq n} |A_k f(x) - \mu(f)| \geq \varepsilon \right\}$, that we call the rate of convergence in Birkhoff theorem.

References

[1] Young L.-S. Statistical properties of dynamical systems with some hyperbolicity // Ann. of Math. 1998. V. 147. P. 585–650.
[2] Rey-Bellet L., Young L.-S. Large deviations in non-uniformly hyperbolic dynamical systems // Ergodic Theory Dynam. Systems. 2008. V. 28. P. 587–612.

[3] Kachurovskii A. G. The rate of convergence in ergodic theorems // Russ. Math. Surveys. 1996. V. 51, N. 4. P. 653–703.

The global fractal method and the fractal paradigm in fundamental radar problems

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The paper discusses the main methodological problems arising from the use of the global fractal method proposed by author, and the existing fractional operators method. The word "method" in the title means that we discuss not just the theory of fractals and fractional differential calculus, but also about their physical and electronic applications. Relevance of the work is primarily based on the crucial need to understanding the fractal and chaos as a single integrated process of the fractal paradigms formation in a broad range of natural and humanitarian sciences, ie, "natural fractal" [1]. Simplifying, we can say that fractals constituted thin amalgam on the powerful science backbone at the end of XX century. In the present situation attempts to downplay their importance and rely only on classical knowledge became the intellectual fiasco. I should note that my ideas about fractals and fractional operators, whom I talked about three decades ago, nowadays already confidently passed from the stage of purely speculative to the stage of tangible reality and reached their maturity as a powerful tool for description of classical and anomalous stochastic processes. The results of our research on fractals in radar,multi-dimensional signal processing and fractal antennas are placed in the "the Report of the Russian Academy of Sciences Presidium. Scientific achievements of Russian Academy of Sciences" in 2007 (p. 41), 2009 (p. 24), 2011 (p. 199) and in the "Report to the Government of the Russian Federation" (2012, p. 242) - see, in particular, in [1, 2] and on the author's website (www.potapov-fractal.com).

References

[1] A. A. Potapov *"Fractal Method and Fractal Paradigm in Modern Natural Science"*, Voronezh, Nauchnaya Kniga, 2012.
[2] A. A. Potapov *"Fractals in Radio Physics and Radar: Sample Topology"*, Moscow, University Library, 2005.

Multistability and transition to chaos in the web map with weak dissipation

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We investigate the effect of weak linear dissipation on the dynamics of degenerate (in terms of KAM-theorem) Hamiltonian systems which are known to have a special structure of phase space called the stochastic web usually [1]. Since the nonlinearly driven harmonic oscillator is known to be the simplest system of this type [1] we consider the nonlinearly driven *dissipative* harmonic oscillator:

$$\ddot{x} - 2\gamma\dot{x} + \omega_0^2 x = -\frac{\omega_0 K}{T}\sin x \sum_{n=-\infty}^{+\infty}\delta(t - nT).$$

We obtained the precise stroboscopic map for this system:

$$x_{n+1} = (x_n \cos\tfrac{2\pi}{q} + (y_n + K\sin x_n + \gamma x_n)\sin\tfrac{2\pi}{q})e^{-\gamma\frac{2\pi}{q}},$$
$$y_{n+1} = ((y_n + K\sin x_n)\cos\tfrac{2\pi}{q} - (x_n(1+\gamma^2) + \gamma(y_n + K\sin x_n)\sin\tfrac{2\pi}{q})e^{-\gamma\frac{2\pi}{q}}. \quad (1)$$

The three relevant parameters of (1) are the dissipation parameter γ, the order of the resonance $q = 2\pi/\omega_0 T$ and the amplitude of the external signal K which is in fact the nonlinearity parameter. We investigated the evolution of the coexisting attractors and their basins with the change of the parameters K and γ for the integer values of q.

We obtained that the number of coexisting attractors grows with the increase of the nonlinearity parameter and diminishes with the increase of the dissipation. Also we revealed that the location of the attractors and the structure of their basins remains practically the same in the wide range of parameters if the nonlinearity and dissipation change simultaneously so that the ratio K/γ is constant.

Also it was shown that the transition to chaos occurs via the rigid transition for the majority of the attractors.

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References

1. G.M. Zaslavsky. Physics of Chaos in Hamiltonian Systems. Imperial College Press, 1998

Structure of the parameter plane for conservatively coupled Hénon maps at different dissipation levels

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Dynamics of systems with small dissipation level is a rather popular object for investigation, especially numerical, in nonlinear dynamics. Usually the investigation is carried out for a low-dimensional systems which are autonomous or periodically driven ones. At the same time the dynamics of coupled systems in the case of small dissipation level is not investigated in such detail. Meanwhile, since both coupled systems and weakly dissipative systems demonstrate a number of very interesting specific phenomena, combining these two classes of systems seems to be a promising way to obtain new interesting features of dynamics. In the present work we are trying to investigate the dynamics of such a system. For this purpose we take a simple example of coupled systems — two coupled Hénon maps

$$x_{n+1} = \lambda_1 - {x_n}^2 - by_n + \varepsilon(x_n - u_n),$$
$$y_{n+1} = x_n, u_{n+1} = \lambda_2 - {u_n}^2 - bv_n + \varepsilon(u_n - x_n), v_{n+1} = u_n,$$

— and look out what is going on with the decrease of dissipation. At infinite level of dissipation this system turns into the system of two coupled logistic maps, which is investigated very well (see, e.g., Juan J.-M. et al. //Phys. Rev. A, 1983, 28, p.1662). It also allows one to vary dissipation level up to the zero.

We have found that the varying of dissipation in the system of conservatively coupled Hénon maps could lead to sufficient complication the transition to chaos. Instead of one-parametric dynamics in widespread domain in the parameter plane in the case of big dissipation one can fall into the region with sufficiently two-parametric dynamics. The Feigenbaum line of transition to chaos becomes divided into three fragments. We also report the existence of the Feigenbaum line fragment with different scaling properties on different sides, namely of H and C type.

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Experimental research of stochastic Andronov-Hopf bifurcation in self-sustained oscillators with additive and parametric noise

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Although the questions concerning the noise influence on the behavior of dynamical systems attract the interest of many researchers, not all noise phenomena are studied in full. Particularly, the stochastic bifurcations still leave many questions. In the presence of noise we can't distinguish the limit sets, which exist in deterministic system; therefore researchers often associate bifurcations in stochastic systems with qualitative changes of probability distribution.

The purpose of this work is experimental researches of the features of stochastic supercritical Andronov–Hopf bifurcation in different systems with different noise characteristics. The researches were based on the consideration of evolution of the probability distribution in the self-sustained oscillators with increasing of noise. Two systems were chosen for studies: Van der Pole and Anishchenko–Astakhov oscillators. The evolution of probability distribution with increase of noise intensity was found for the both systems in case of additive noise and in case of noisy modulation of a generation parameter. Results of the physical experiments for analog circuit are compared with the results of a computer simulation. Experiments confirm the theoretical conclusions about bifurcation interval existence for the supercritical Andronov–Hopf bifurcation in Van der Pole self-sustained oscillator with additive noise [1]. We can't speak about bifurcation interval in Anishchenko–Astakhov self-sustained oscillator, but the postponement of bifurcation takes place. Postponement of the stochastic bifurcation is observed also in Van der Pole and Anishchenko–Astakhov oscillators with parametric noise. In all considered cases the increase of noise level leads to disappearance of closed crater in distribution, which corresponds to noisy limit cycle. This can be interpreted as the destruction (suppression) of self-sustained oscillations by noise action.

[1] W. Ebeling, H. Herzel, W. Richert, L. Schimansky-Geier, Influence of noise on Duffing-van der Pol oscillators// Zeischrift f́ur angewandte Vathematik und Mechanik

Types of homoclinic trajectories in one-dimensional dynamics

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We consider the problem of coexistence periodic and homoclinic trajectories of dynamical systems generated by continuous interval maps. What will be the solution of this problem depends on what classification of homoclinic trajectories we will use. It is natural to classify homoclinic trajectories using classification of their limit sets.

If cycles are distinguished by the period, then the following linear ordering of natural numbers set

$$1 \triangleright 3 \triangleright 5 \triangleright 7 \triangleright 9 \triangleright \ \dots \ \triangleright 2{\cdot}1 \triangleright 2{\cdot}3 \triangleright 2{\cdot}5 \triangleright \ \dots \ \triangleright 2^2{\cdot}1 \triangleright 2^2{\cdot}3 \triangleright 2^2{\cdot}5 \triangleright \ \dots$$

describes the coexistence of homoclinic trajectories to cycles of different periods [1].

If cycles are distinguished by the types (i.e., the cyclic permutations generated by restriction of the map over the cycles), then new aspects of our problem appear and, in particular, we discuss: 1) the type of cycle such that any map has homoclinic trajectory to a cycle of this type, 2) the approximation of homoclinic trajectories by cycles of certain types [2].

We also consider the problem for some special classes of n-dimensional continuous maps [3]. In this case, we distinguish cycles of period p by the $(n \times p)$-matrixes, which are the generalizations of the cyclic presentations of the types of one-dimension cycles.

References

[1] Fedorenko V.V., Sharkovsky A.N. *Stability properties of dynamical systems to have homoclinic trajectory*, in *Oscillation and stability of solutions of differential-functional equations*, Institute of Mathematics, Academy of Sciences of UkrSSR, Kiev, 1982, 111–113 [in Russian].

[2] Fedorenko V.V., Sharkovsky A.N. *On coexistence of homoclinic and periodic trajectories*, Nonlinear Dynamics *6* (2010), 207–217 [in Russian].

[3] Fedorenko V.V., Sharkovsky A.N. *Homoclinic trajectories in one-dimensional dynamics*, Journal of Difference Equations and Applications *18* (2012), 579–588.

Structure of components of chain recurrent set

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Results obtained by various authors, describe the topological structure of attractors of a flow. It will be presented that : the attractor of flow on topological manifolds have the shape of a finite polyhedron [1], [2] ; the inclusion of the global attractor into the state space is a shape equivalence [3]. According to these results, spaces like solenoids or Hawaiian earning cannot be attractors of a flow. For flows defined on a compact metric space, it will be shown in the paper that the connectivity components of a chain recurrent set possess a stronger connectivity known as joinability [4] (or pointed 1-movability in the sense of Borsuk) , an invariant of strong shape [5]. For example Hawaiian earning is joinable, and the space consisting of sin (1/x) curve together with its limit points is joinable, while solenoids are not and cannot be components of the chain recurrent set .

References

[1] S.A. Bogatyi and V.I. Gutsu, On the structure of attracting compacta, Differentsialnye Uravneniya, 25 (1989), no. 5, 907–909.

[2] B. Giinther, J. Segal, Every attractor of a flow on a manifold has the shape of finite polyhedron, Proc. Amer. Math. Soc. 119 (1993), 321-329.

[3] Kapitanski, Rodnianski, Shape and Morse theory of attractors, Communications on Pure and Applied Mathematics Volume 53, Issue 2, pages 218–242, February 2000

[4] Piotr Minc, Remainders of arcwise connected compactifications of the plane, Top. Appl. Volume 154, Issue 8, 15 April 2007, Pages 1592–1599

[5] N. Shekutkovski, Intrinsic definition of strong shape for compact metric spaces, Topology Proceedings (2012) 39, p. 27-39.

Intrinsic shape in dynamical systems

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The interaction between topology and dynamical systems will be considered through shape theory tools. Namely, using the intrinsic approach to shape, the notion of proximate net will be introduced as a sequence of near continuous functions which converge in the homotopic sense and natural way of producing one in a given flow will be discussed. The central part of the talk will be to generalize the shape-attractor theorem for Morse decomposition and to give the strong shape version using the intrinsic approach.

Non-integrability of the Swinging Atwood's Machine using higher order variational equations.

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Non-integrability criteria, based on differential Galois theory and requiring the use of higher order variational equations, are applied to prove the non-integrability of the Swinging Atwood's Machine for values of the parameter which can not be decided using first order variational equations.

This is a joint work with R. Martínez. See Discrete and Continuous Dynamical Systems A **29** (2011), 1–24.

Coherence resonance and traveling waves regimes destruction in model of an active medium with periodic boundary conditions

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The model of an active medium with periodical boundary conditions, which cell is represented by the FitzHugh-Nagumo oscillator (simplified neuron model) [2], is studied. Such systems are used to model processes occurring in neural tissues [1,3]. The element of medium can demonstrate either the excitable and self-oscillatory behavior in dependence of the control parameters values. Irrespective of the elementary cell regime the phenomenon of phase multi-stability, which consists in coexisting of traveling waves regimes with various wavelengths, is observed in medium without noise presence. The noise influence on the active medium wave regimes is studied, the comparison between spatially uncorrelated and spatially uniform noise impacts is carried out. It is shown that uniform noise leads to destruction of the wave regime and transition to uniform oscillations mode much faster (for lesser values of noise intensity) then uncorrelated one. Both in the case of uniform noise and in the case of uncorrelated one the phenomenon of coherence resonance is observed. But the optimal level of noise for coherence resonance in medium is considerably differed from one in single FitzHugh-Nagumo oscillator. It is assumed that coherence resonance is concerned with traveling waves regimes destruction.

References

[1] Ermentrout B., Pinto D. Neurophysiology and waves // SIAM News, 2007, vol. 40, no. 2.
[2] FitzHugh R.A. Impulses and physiological states in theoretical models of nerve membrane // Biophysical Journal, 1961, vol. 1, pp. 445-466.
[3] Lancaster J. L., Hellen E.H., Leise E.M. Modeling excitable systems: Reentrant tachycardia // American Journal of Physics, 2010, vol. 78, no. 1, pp. 56-63.

Dynamical analysis of two models of cancer tumor growth with immunotherapy

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We study dynamical properties of two cancer tumor growth models using a localization analysis of compact invariant sets [1,2]. The first one is a 4-dimensional model of the superficial bladder cancer tumor growth for which the Bacillus Calmette-Guérin (BCG) immunotherapy is applied [3]. Our main results are as follows: 1) we derive formulas for bounds of the polytope in the open positive octant which contains all compact invariant sets; 2) we show the existence of the bounded positive invariant domain in closed positive octant; 3) we obtain sufficient conditions for the attraction of any point in the open positive octant to the tumor-free equilibrium point (the global asymptotic tumor clearance theorem). Our second model is a 7-dimensional tumor growth model under conditions when immunotherapy and cancer vaccination are applied [4]. Our main results are as follows: 1) we derive formulas for bounds of the polytope in the open positive octant which contains all compact invariant sets; 2) we show the existence of the bounded positive invariant domain in the closed positive octant; 3) some results concerning dynamics around the tumor-free equilibrium point are described as well. Biological implications are discussed.

References

1. Krishchenko A.P., Starkov K.E., Phys. Lett. A, 2006; 353: 383-388.

2. Starkov K.E., Coria LN., Nonlinear Analysis: Real World Application, 2013, v.14: 1425-1433.

3. Bunimovich-Mendrazitsky S., Shoshat E., Stone L., Bull. Math. Biology, 2007, 69, 1849-1870.

4. Joshi, B., Wang, X., Banerjee, S., Tian, H., Matzavinos, A, Chaplain, M.A.J., J. Theor. Biology, 2009, v.259, 820–827.

One parameter analysis of Homoclinic Ω-explosion: intervals of hyperbolicity and their boundaries.

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At the bifurcation boundary separating Morse-Smale systems from systems with complicated dynamics there are systems with homoclinic tangencies. Moreover, when crossing this boundary, infinitely many periodic orbits appear immediately, just by “explosion”. This fact was established by Gavrilov and Shilnikov in 1973. Newhouse and Palis have shown that in this case there are infinitely many intervals of values of the splitting parameter corresponding to hyperbolic systems. In the present paper, we show that such hyperbolicity intervals have natural bifurcation boundaries, so that the phenomenon of homoclinic Ω-explosion gains, in a sense, complete description in the case of two-dimensional diffeomorphisms. We prove that the loss of hyperbolicity is connected with the appearance of homoclinic tangencies, nontransversal heteroclinic cycles or saddle-node periodic orbits.

Functionalization of a parameter in a saddle-node bifurcations

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The paper discusses the problem of sufficient criteria for saddle-node bifurcations in dynamical systems. Such problems can be presented in different ways as the operator equation

$$y = B(\mu)y + b(y,\mu) + u(\mu), \tag{1}$$

where $y \in R^n$, μ - a scalar or vector parameter, $B(\mu)$ - a square matrix, the nonlinearity $b(y,\mu)$ with quadratic terms in y, and the vector $u(\mu)$ satisfies $u(\mu_0) = 0$ for some μ_0. The operator $B(\mu_0)$ has eigenvalue 1. Under these conditions, equation (1) with μ close to μ_0 in the neighborhood of $y = 0$ can has continuous branch of solutions $y = y^*(\mu)$ such that $y^*(\mu_0) = 0$.

One of the effective methods for investigation of bifurcation is the method of parameter functionalization proposed by M.A. Krasnoselsky. This method was used to study the transcritical bifurcation, Andronov-Hopf pitchfork bifurcation. However, until now the method of parameter functionalization was not used for investigation of saddle-node bifurcations. This is due to the fact that this bifurcation is not an equation of the form $y = F(y, \mu)$ with the function F, with the property $F(0, \mu) \equiv 0$.

This report provides a justification of the method of parameter functionalization for saddle-node bifurcations, discusses some applications to the analysis of bifurcation phenomena in dynamical systems.

Expanding Baker Maps as models for dynamics emerging for 3D homoclinic bifurcations

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At the beginning of this century families of limit return maps associated to the unfolding of 3D-homoclinic tangencies have been constructed. In the case in which the unstable manifold of the saddle point involved in the homoclinic tangency has dimension two, many different strange attractors have been numerically observed for the corresponding family of limit return maps. Moreover, for some special value of the parameter, the respective limit return map is conjugate to what we call bi-dimensional tent map. This piecewise affine map is a nice example of what we call Expanding Baker Map (EBM), and the main objective of this talk is to present how many of the different attractors exhibited for the limit return map resemble the ones observed for EBM. To this end, a special one parameter family of EBM will be selected, being this family the best choice in a suitable dynamical sense.

This is a joint work with A. Pumariño, J. A. Rodríguez and E. Vigil.

Shadowing in actions of abelian and nonabelian groups

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We introduce notion of shadowing property for actions of finitely generated not necessarily abelian groups. In contract with shadowing for diffeomorphisms and flows we show that shadowing property depends not only on hyperbolicity but on the group structure as well.

For nilpotent groups we prove analog of the shadowing lemma. We give an example of an action of a solvable group, whose shadowing property depends on quantitative properties of hyperbolicity. Finally we prove that there is no linear action of free nonabelian group which has shadowing property.

The talk is based on joint work with A. Osipov and S. Yu. Pilyugin.

Lipschitz inverse shadowing for nonsingular flows

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The theory of shadowing is now a well-developed part of the modern global theory of dynamical systems. In particular, it is known that Lipschitz shadowing is equivalent to structural stability for diffeomorphisms and for flows (see [1,2]).

Last years there was an intense development of the inverse shadowing problem. The property of inverse shadowing means that it is possible to approximate any exact trajectory by a pseudotrajectory from a specified class.

It was proved that structural stability implies Lipschitz inverse shadowing for diffeomorphisms (see [3]) and for flows (see [4]).

I will explain how one can prove that Lipschitz inverse shadowing for diffeomorphisms is equivalent to structural stability. Also I will show how one can prove a similar for nonsingular flows.

References

[1] S. Yu. Pilyugin and S. B. Tikhomirov, "Lipschitz Shadowing Implies Structural Stability," Nonlinearity 23, 2509-2515 (2010).

[2] K. Palmer and S. Yu. Pilyugin and S. B. Tikhomirov, "Lipschitz shadowing and structural stability of flows,"

[3] S. Yu. Pilyugin, "Inverse Shadowing by Continuous Methods," Discrete Contin. Dyn. Syst. 8, 29-38 (2002).

[4] Y. Han and K. Lee, "Inverse shadowing for structurally stable flows," Dyn. Syst. 19, no. 4, 371-388 (2004).

Lorenz attractors for flows and diffeomorphisms

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We discuss a generalisation of the classical theory of Lorenz attractors based on two notions: volume-hyperbolicity and chain-transitivity. The theory extends to time-periodic perturbations of systems with a Lorenz attractor, to lattices of weakly coupled systems of this type, and other examples. We show that Newhouse wild hyperbolic sets and Bonatti-Diaz blenders can be typical constituents of higher-dimensional analogues of the Lorenz attractor. We also discuss local and global bifurcations that create Lorenz attractors and their higher-dimensional analogues.

Contact transformations and normal forms in thermodynamics of non-ideal media

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We study the critical phenomena in non-ideal media by considering the perturbations of model thermal equations of state, normalized with respect to contact transformations that preserve the Gibbs-Duhem equation. For this purpose, we use a method of generalized normal forms (see [1]).

First, consider the equation of state of a mixture of non-ideal gases, which is often written in the form of a *virial expansion*:

$$P = T\sum_{i=1}^{k} n_i + \sum_{|m|\geq 2} B_m n^m, \qquad m = (m_1, \ldots, m_k), \tag{1}$$

where n_i denotes the concentration of the i-th component and the *virial coefficients* B_m depend only on T. We prove the following

Theorem [1, Th 3]. *Let B_m be analytic functions of temperature near $T = 0$. Then for each integer $M \geq 2$, all the virial coefficients B_m with $|m| \leq M$ in expansion* (1) *can be eliminated by a contact transformation.*

Next, consider the Debye-Hückel model for multicomponent plasma:

$$P = T \sum_{i=1}^{k} n_i - \frac{\sqrt{\pi}}{3\, T^{1/2}} \Big(\sum_{i=1}^{k} n_i\, q_i^2 \Big)^{3/2}, \tag{2}$$

where q_i denotes the electric charge of a particle of the i-th type. We classify the perturbations of (2) in the sense described above (see [1, Th 4]) and apply this result to analize the hypothetical plasma phase transition in non-ideal hydrogen plasma.

This research is supported by the Chebyshev Laboratory (Mathematics and Mechanics Faculty, St Petersburg State University) under RF Government grant 11.G34.31.0026.

References

[1] Vaganyan A S (2013) Generalized normal forms of infinitesimal symplectic and contact transformations in the neighborhood of a singular point, arXiv:1212.4947 (preprint).

On H-theorem for Liouville Equation, Vlasov Dynamics and Boltzmann extremals

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We describe connections between Liouville Equation, Vlasov Dynamics, Hydrodynamics and Magneto-hydrodynamics ([1-7]). We consider H-theorem, Boltzmann Extremes and asymptotic behavior as time tends to infinity, and the theorem "Time Averages coincides with Boltzmann Extreme"[5], Arnold-Kozlov lemma on commuting fields with a modification for Vlasov-Maxwell and Vlasov-Poisson equations, derivation of Hamilton-Jacobi equations from Liouville and Vlasov equations.

References

[1] L.D. Landau and E.M. Lifshits. In: Theoretical Physics Vol. 2, Nauka, Moscow (1967).
[2] V. V. Vedenyapin Boltzmann and Vlasov Kinetic Equations, (in Russian) FIZMATLIT, Moscow, 2001.

[3] V. V. Kozlov, "The generalized Vlasov kinetic equation Russian Math. Surveys, 63:4 (2008), 691-726.
[4] V.V.Vedenyapin, M.A.Negmatov. Derivation and classification of Vlasov-type and magneto-hydro-dynamics equations: Lagrange identity and Godunov's form.Teoret. Mat. Fiz., 170:3 (2012), 468-480.
[5] V.V.Vedenyapin. Time Averages and Boltzmann Extremals. Soviet Math. Doklady,v. 422, N2, 2008,pp.161-163.
[6] V.V.Vedenyapin, A.V.Sinitsyn and E.V.Dulov. Boltzmann and Vlasov Kinetics. Amsterdam-London-New-York-Tokio, Elsvier, 2011. Pp.1-304.
Izv. Akad. Nauk SSSR Ser. Mat., 42:5 (1978), 1063-1100.
[7] Boltzmann L. Weitere Studien uber das Warmegleichgewicht unter Gasmolekulen. Wien: Akad. Sitzungsder, 1872. Bd. 66. S. 275-370.

Dynamics of Piecewise Translations

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Dynamics of Piecewise Translations is a new emerging area in dynamical systems. They have some features of area-preserving maps, at the same time exhibiting new interesting phenomena. Their areas of application include random fields and computer science.

A piecewise translation is defined as follows. Let B be a region in $\mathbb{R}^n$, take a partition $B = B_1 \sqcup \cdots \sqcup B_r$, and fix some vectors $v_1, \ldots, v_r$. Finally, let $F|_{B_i} \colon x \mapsto x + v_i$, $i = 1 \ldots r$, provided it is a well-defined map $F \colon B \to B$. We are interested in the limit behavior of the dynamical system defined by F on B.

For the number of pieces $r \leq n$, the problem reduces to the smaller number of dimensions or becomes trivial. We will see that for $r = n + 1$, the system stabilizes after finitely many iterates, regardless of the partition. The emerging attractor has positive Lebesgue measure, and the induced dynamics is a region exchange transformation. If the translation vectors v_i are generic, then the attractor admits a nice geometric description. These results are joint with A. Gorodetski and S. Northrup.

We will also discuss the case of $r > n + 1$ and related open problems.

References

[1] R. L. Adler, T. Nowicki, G. Świrszcz, and C. Tresser. Convex dynamics with constant input. *Ergodic Theory Dynam. Systems*, 30(4):957–972, 2010.

[2] M. Boshernitzan and I. Kornfeld. Interval translation mappings. *Ergodic Theory Dynam. Systems*, 15(5):821–832, 1995.

[3] J. Buzzi. Piecewise isometries have zero topological entropy. *Ergodic Theory Dynam. Systems*, 21(5):1371–1377, 2001.

[4] D. Volk. Almost every interval translation map of three intervals is finite type. *arXiv:1203.3405*, 2012.

[5] M. Welling. Herding dynamic weights for partially observed random field models. In *In Proc. of the Conf. on Uncertainty in Artificial Intelligence*, 2009.

Symbolic tools for exploration of deterministic chaos

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Computational technique based on the symbolic description utilizing kneading invariants is proposed for explorations of parametric chaos in a two exemplary systems with the Lorenz attractor: a normal model from mathematics, and a laser model from nonlinear optics. The technique allows for uncovering the stunning complexity and universality of the patterns discovered in the bi-parametric scans of the given models and detects their organizing centers – codimension-two T-points and separating saddles.

Dynamics of point-wise and diffusive models of the iodine-xenon oscillations

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We investigate point-wise and circular diffusive models of iodine-xenon oscillations in thermal nuclear reactors. Stability conditions for stationary solution of the point-wise model of the iodine-xenon oscillations are derived. It is also shown that the bifurcation of stability loss in the point-wise model

of iodine-xenon oscillations is sharp. Besides, the circular diffusive model of iodine-xenon oscillations is examined.

Models of cell cycle dynamics and clustering

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Motivated by experiments and theoretical work on respiratory oscillations in yeast cultures, we study phenomenological ordinary differential equations models of the cell cycles of large numbers of cells, with cell-cycle dependent feedback. We assume very general forms of the feedback and study the dynamics, particularly the clustering behavior of such systems.

Biologists have long observed periodic-like oxygen consumption oscillations in yeast populations under certain conditions. We hypothesized that certain of these oscillations could be accompanied and/or caused by a weak form of cell cycle synchronization that we call clustering ([Robertson 2008], [Boczko 2010], [Young 2012]). We developed some novel ODE models of the cell cycle. We gave proofs and simulations showing that both positive and negative feedback are possible agents that can cause clustering of populations within the cell cycle for these models. Furthermore, this clustering phenomenon was seen to be robust; it occurs for a variety of models, a broad selection of parameter values in those models and even for random and stochastic perturbations of the models. Since there are necessarily an integer number of clusters, clustering can lead to periodic-like behavior with periods that are nearly integer divisors of the period of the cell cycle. Related experiments have shown conclusively that cell cycle clustering occurs in oscillating cultures [Stowers 2012].

In this talk we discuss recent progress in the study of the mathematical models and the implications of the mathematical results on the biology of the cell cycle. In particular we discuss the effects of the time scale of diffusion of metabolites across cell membranes effect solution of the models in a way that makes the phenomenon of clustering more robust in many cases.

Operator methods in the problem of constructing Arnold tongues

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We consider discrete time systems

$$x_{n+1} = F(x_n, \mu), \quad n = 0, 1, 2, ..., \quad x_n \in R^N \,, \tag{1}$$

where μ is a vector parameter, the function $F(x, \mu)$ is continuously differentiable. Let $F(0, \mu) \equiv 0$. Systems (1) has the equilibrium $x = 0$ for all μ.

Suppose that, for some $\mu = \mu_0$, the numbers $e^{\pm 2\pi\theta_0 i}$, where $0 < \theta_0 \leq \dfrac{1}{2}$ and θ_0 rationally, are eigenvalues of the matrix $A(\mu_0) = F'_x(0, \mu_0)$. In this case, for values of μ that are close to μ_0, the emergence of periodical solutions in the neighborhood of the equilibrium point $x = 0$ in the system (1) is possible.

We study Arnold tongues: the sets of parameter values for which the small-amplitude periodic orbits (near an equilibrium) exist; the Arnold tongues have the form of narrow beaks. We present the basic postulates of a new method for studying Arnold tongues. We describe the sets of parameter values for which the small-amplitude q-periodic trajectories of the system (1) exist for a fixed q. We construct the approximate formulae for the cycles of the system (1) and the corresponding values of the parameters. The method makes it possible to detect bifurcation parameter values; it leads to an iteration procedure for approximately studying problems depending on many parameters. Applications to the theory dynamical systems are also discussed.

On the origin of birhythmicity in ensembles of coupled oscillators

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Large assemblies of coupled oscillators are often polyrhytmic; for example, recorded extracellular oscillations of human neurons demonstrate alternating epochs of fast and slow oscillations. To reproduce this phenomenon, most of the existing models involve oscillating elements whose intrinsic timescales strongly differ. Here, we discuss a mechanism which ensures birhytmicity in ensembles where all elements share the intrinsic timescale. Albeit non-generic, considered families of dynamical systems share typical properties of many existing models in technics and neuroscience. We consider networks built of oscillatory units with the same eigenfrequency; coupling terms in the governing equations are proportional to velocities of the elements. No restrictions are imposed either on the symmetry of the coupling or on its pattern (small world, mean field, next neighbors, pairwise or triple interaction etc.).

In the parameter space of the ensemble, destabilization of the equilibrium occurs by means of the Andronov-Hopf bifurcation. On the large part of the stability boundary, the spectrum of the linearized flow contains not one (as usually) but two pairs of purely imaginary eigenvalues. Within this context the so-called "double Hopf" bifurcation becomes a codimension-one phenomenon. Of the two resulting critical frequencies, one is typically much lower than the individual frequency of an element, whereas the other one is distinctly higher. Accordingly, in the nonlinear regime the ensembles are potentially capable of performing both slow and fast modes of oscillations. We illustrate this general phenomenon by numerical data obtained from ensembles of oscillators with different coupling patterns and demonstrate that after the transition the system can possess two stable limit cycles (respectively, one "fast" and one "slow") or a quasiperiodic state.

Shadowing of non-transversal heteroclinic chain

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Our goal is to present an abstract result which explains the construction of 'diffusing' orbits for nonlinear Schrodinger equation from [1] (see also [2]) from geometric point of view.

In the simplest situation the problem can be stated as follows: we have a heteroclinic chain of connections between periodic orbits, but the connections are not transversal, nor are the periodic orbits hyperbolic. In general such chain can not be shadowed (followed) by an orbit. However, in the context of Galerkin projections of nonlinear Schrodinger equations this can be achieved.

Hopefully, this will clarify the geometry behind the phenomenon and will make it easier to apply it to other systems.

This is a joint work with A. Delshams and A. Simon.

References

[1] J. Colliander, M. Keel, G. Staffilani, H. Takaoka, T. Tao, *Transfer of energy to high frequencies in the cubic defocusing nonlinear Schrödinger equation*, Invent. Math. (2010) 181:39-113.

[2] M. Guardia, V. Kaloshin, *Growth of Sobolev norms in the cubic defocusing nonlinear Schrödinger equation*, preprint.

Regimes of genetic structure and number dynamics in evolutionary model of two-aged population

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In the present study an evolution of population with two age groups is considered on the model level and more attention has given to consideration of inheritance mechanisms for population characters. A mathematical model of the dynamics of genetic structure together with age groups sizes is developed for population with genetically defined survival rate of its reproductive part. This model with analogous one for population with genetically defined reproductive potential allows us to construct a fuller appreciation of natural evolution in structured population.

Conducted investigation in general confirms results of previous investigations that consider dynamics of age groups numbers only. So an increase in the reproductive potential and survival rate is accompanied by complication of the population number dynamics. So there is large variety of population dynamics in the model; from stable dynamics of population number and its genetic structure to regular fluctuations and chaotic ones. One can see various fantastic forms of strange attractors with various dimensions, which is calculated by Lyapunov exponents in our work. Alsow we find that evolution increase of considered population parameters may be very non-monotonic with serious fluctuation too. There is large variety of dynamic regimes of population genetic structure in considered models. And increase in the average survival rate of the reproductive part may both destabilise and stabilise the genetic compositions of the age groups in the populations.

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Counterexamples to the regularity of Mane projections and global attractors

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We study the global attractors of abstract semi-linear parabolic equations and their projections to finite-dimensional planes. It is well-known that the attractor can be embedded into the finite-dimensional inertial manifold if the so-called spectral gap condi-tion is satisfied.

We show that in the case when the spectral gap condition is violated, it is possible to construct a nonlinearity in such way that the corresponding attractor cannot be embedded into any finite-dimensional Log-Lipschitz manifold and, therefore, does not possess any Mane projections with Log-Lipschitz inverse. In addition, we give an example of finitely smooth nonlinearity such that the attractor has finite Hausdorff but infinite fractal dimension.

Branched coverings and pseudo-Anosov homeomorphisms

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Explicit construction will be presented, by which, beginning with the (generalized) pseudo-Anosov homeomorphism with non-orientable invariant foliations, we obtain pseudo-Anosov homeomorphism with orientable invariant foliations. This homeomorphism is defined on a surface which is two-fold branched covering of the original one and covers the original homeomorphism.

The construction is interesting because it allows one to build new examples of pseudo-Anosov homeomorphisms and is used to establish some arithmetical properties of dilatation (coefficient of expansion/contraction of invariant foliations) of generalized pseudo-Anosov homeomorphisms. Recall that the dilatation is logarithm of topological entropy and it is the same for the original homeomorphism and its lift.

This construction may be extended to diffeomorphisms of surfaces with one-dimensional hyperbolic attractors.

Transverse equivalence of complete conformal foliations

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Recent results will be presented on the investigation of the structure of complete conformal foliations of codimension $q \geq 3$.

Let $(M, \mathcal{F})$ be an arbitrary smooth foliation. Remind that a subset of a manifold M is called a saturated whenever it is the union of some leaves of a foliation $(M, \mathcal{F})$. By definition, an attractor of a foliation $(M, \mathcal{F})$ is nonempty saturated subset $\mathcal{M}$, if there exists a saturated open neighborhood $Attr(\mathcal{M})$ such that the closure of every leaf from $Attr(\mathcal{M}) \setminus \mathcal{M}$ includes $\mathcal{M}$. Here $Attr(\mathcal{M})$ is named as an attractor basin. If an addition $M = Attr(\mathcal{M})$, then the attractor $\mathcal{M}$ is called global.

Further we assume that the codimension of conformal foliations is $q \geq 3$.

As it was proved by the author [1], every conformal foliation $(M, \mathcal{F})$ either is Riemannian or has an attractor that is a minimal set of $(M, \mathcal{F})$, and the restriction of the foliation to the attractor basin is a $(Conf(S^q), S^q)$-foliation.

We proved that every conformal foliation $(M, \mathcal{F})$ on a compact manifold M either is a Riemannian foliation or a $(Conf(S^q), S^q)$-foliation with a finite family of minimal sets. They all are attractors of this foliation [1].

If $(M, \mathcal{F})$ is a complete non-Riemannian conformal foliation, it is a $(Conf(S^q), S^q)$-foliation and has global attractor $\mathcal{M}$ [2]. A notion of the global holonomy group is defined for such foliation $(M, \mathcal{F})$.

We decide the problem of the classification of conformal foliations with respect to transverse equivalence. We prove that two conformal foliations $(M_1, \mathcal{F}_1)$ and $(M_2, \mathcal{F}_2)$ are transversally equivalent if and only if their global holonomy groups Ψ_1 and Ψ_2 are coincided (up to conjugation in the Lie group $Conf(S^q)$). By other words, the global holonomy group is the complete invariant of the class of transversally equivalent conformal foliations.

Therefore every two of transversally equivalent conformal foliations have global attractors with the same transverse structure.

Examples of conformal foliations with exceptional and exotic global attractions are constructed.

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References

[1] Zhukova N. I., Attractors and an analog of the Lichnerovich conjecture for conformal foliations, Siberian Mathematical Journal. (2011) **52**:3, 436–450.
[2] Zhukova N. I., Global attractors of complete conformal foliations, Sbornik Mathematics. (2012) **203**: 3 , 308—405.

Interaction of oscillating dissipative optical solitons

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We show that a transition of stationary light pulses into oscillating ones via an Andronov-Hopf bifurcation in nonlinear fiber cavity with external coherent injection can lead to a drastic enhancement of their weak interaction and formation of new types of pulse bound states amenable to experimental observation. These bound states are characterized by different distances between the pulses as well as oscillation phase differences, i.e., correspond to different pulse synchronization regimes.

Теория прямых измерений в квантовой механике

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Прямые измерения затрагивают область экспериментальной физики в квантовой механике. Интерес к теоретическим исследованиям в этой области восходит к именам Боголюбова, Колмогорова, Блохинцева и др. (см. [1,2,3]). Прямые измерения в непрерывном фазовом пространстве и непрерывном времени являются здесь началом длинного пути. Первые шаги заключены в локализации прямых измерений в фазовом пространстве. Второй шаг связан с необходимостью дискретизации времени, желательно в периодической форме, чтобы сохранить спектроскопические исследования (см. [4]).

С точки зрения фундаментальной математики это - принципиальный вопрос: каким образом можно переходить от линейных уравнений математической физики к нелинейным моделям. Ниже излагается первый систематический подход к этой проблеме.

В.И. Арнольд редко пояснял свои изыскания, в частности, волновые фронты (см. [5]). Мы получили возможность расшифровать эти понятия. Это мотивировано теорией прямых измерений, которую хотел видеть развитой А.Н. Колмогоров, но мы в состоянии это сделать только сейчас. Конечно этот факт имеет тривиальную причину: в этом случае генератор геометрии фазового пространства равен тождественно нулю, т.е. имеет степень вырождения равную бесконечности.

В квантовой механике, восходящей к работам начала XIX столетия М. Планка (см. [6]), противоречия с измерительной физикой наиболее очевидны. Наиболее парадоксальным примером является отсутствие времени, описывающего динамику во времени впрямую. Хотя во всех учебниках фигурирует $V(x,t)$ тем не менее в примерах $V(x,t) \equiv V(x)$. Это связано с неудобством операторного метода, лежащего в основе квантовой механики, по сравнению с теорией прямых измерений. Сегодня в начале XXI веке мы отчетливо понимаем, что в научной литературе, неявно содержится описание переходных процессов в составляющих малых временах исполнения и многих массах участвующих частиц (см. [7]).

В качестве контраста можно назвать задачи астрофизики, где исполнительные времена и масштабы весьма велики, за исключением моментов столкновений. В квантовой механике условия протекания процессов весьма коротки и в результате взаимодействия среды и пробной частицы (иногда волны) весьма отличаются от наших интуитивных представлений.

В теории прямых измерений мы имеем возможность отправляться от рассмотрения линейных функционалов, вместо традиционных квадратичных. Это естественно связано с геометрией фазового пространства, которая меняется во времени. Геометрия фазового пространства наиболее естественно и необратимо возникает в теории относительности Эйнштейна. Но в теории прямых измерений в квантовой механике размеры и времена процессов могут быть очень малы (см. [8]).

С точки зрения фундаментальной математики теория бифуркаций динамических систем в ее полном развитии позволяет решать проблемы. В частности растяжку масштабов в окрестности критических значений параметров (см. [5]).

Теория прямых измерений, отправляясь от линейных функционалов, дополняет традиционную теорию с непрерывным временем до дискретного времени. Здесь она имеет пересечение с идеями Д.И. Блохинцева, В.И. Арнольда и др. (см. [6,7]).

Литература

1. Боголюбов Н.Н. Собрание научных трудов: в 12т. T.S.: Механика, 1939-1980/ ред. И.И. Плакида, А.Д. Суханов - 2006, 804 с.

2. Колмогоров А.Н. Юбилейное издание в 3-х книгах. Книга 1. Истина - благо. Библиография/ Ред.-совет Ширяев . - М: ФИЗМАТ ЛИТ, 2003, -384с.

3. Ландау Л.Д., Лифшиц Е.М. Квантовая механика (нерелятивистская теория). - Издание 6-е, исправленное. - М.: Физматлит, 2004. - 800с. - ("Теоретическая физика том III).

4. Богданов Р.И. Нелинейные динамические системы на плоскости и их приложения. - М.: Вузовская книга, 2003. 376 с.

5. Арнольд В.И.. Особенности каустик и волновых фронтов (Библиотека математика. Вып. 1). М.:ФАЗИС, 1996.

6. Планк М. Избранные труды. Термодинамика. Теория излучения и квантовая теория. Теория относительности. Статьи и речи.- М.: "Наука 1975, 788с.

7. Блохинцев Д.И. Избранные труды в 2т. Т.2.-М.: Физматлит, 2009, 744с.

8. Нейман Дж. Математические основы квантовой механики. М.: Наука, 1964.

Об особенностях асимптотик решений сингулярно возмущенных задач с кратным корнем вырожденного уравнения

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Асимптотики погранслойных решений сингулярно возмущенных задач в случае кратного корня вырожденного уравнения имеют качественные отличия от асимптотик в случае простого корня. Их можно увидеть на примере краевой задачи

$$\varepsilon^2 u'' = f(u, x, \varepsilon), \quad 0 < x < 1 \tag{1}$$

$$u'(0, \varepsilon) = u'(1, \varepsilon) = 0, \tag{2}$$

где $\varepsilon > 0$ — малый параметр, f — достаточно гладкая функция, имеющая вид

$$f(u, x, \varepsilon) = h(u, x)(u - \varphi(x))^k - \varepsilon f_1(u, x, \varepsilon). \tag{3}$$

Пусть $\overline{h}(x) := h(\varphi(x), x) > 0$ на отрезке $[0; 1]$. Если $k = 1$, то вырожденное уравнение $f(u, x, 0) = 0$, получающееся из (1) при $\varepsilon = 0$, имеет простой (однократный) корень $u = \varphi(x)$, и хорошо известно, что задача (1) имеет для достаточно малых ε погранслойное решение $u(x, \varepsilon)$ с асимптотикой вида

$$u(x, \varepsilon) = \overline{u}(x, \varepsilon) + \Pi(\xi, \varepsilon) + \widetilde{\Pi}(\widetilde{\xi}, \varepsilon), \tag{4}$$

где $\overline{u}(x, \varepsilon) = \varphi(x) + \sum\limits_{i=1} \varepsilon^i \overline{u}_i(x)$ — регулярная часть асимптотики, $\Pi(\xi, \varepsilon) = \sum\limits_{i=1} \varepsilon^i \Pi_i(\xi)$, $\xi = \frac{x}{\varepsilon}$ — левый пограничный ряд (в окрестности точки $x = 0$), $\widetilde{\Pi}(\widetilde{\xi}, \varepsilon)$, $\widetilde{\xi} = \frac{1-x}{\varepsilon}$ — аналогичный правый пограничный ряд (в окрестности точки $x = 1$).

Если $k = 2$, то корень $u = \varphi(x)$ вырожденного уравнения является двукратным, и, как оказалось, существенную роль в вопросе о существовании погранслойного решения задачи (1) играют теперь члены порядка ε, входящие в выражение (3), а именно, функция $\overline{f}_1(x) := f_1(\varphi(x), x, 0)$.

Если $\overline{f}_1(x) > 0$ на отрезке $[0; 1]$, то для достаточно малых ε задача (1) имеет решение с погранслойной асимптотикой вида (4), но качественное отличие от случая простого корня состоит в том, что регулярная часть асимптотики является теперь рядом по степеням $\sqrt{\varepsilon}$ (а не ε, как в случае простого корня): $\overline{u}(x, \varepsilon) = \varphi(x) + \sqrt{\varepsilon}\overline{u}_1(x) + \cdots$, погранслойные переменные ξ и $\widetilde{\xi}$ имеют теперь другой масштаб: $\xi = \frac{x}{\varepsilon^{3/4}}$, $\widetilde{\xi} = \frac{1-x}{\varepsilon^{3/4}}$, а ряды $\Pi(\xi, \varepsilon)$ и $\widetilde{\Pi}(\widetilde{\xi}, \varepsilon)$ являются рядами по степеням $\varepsilon^{1/4}$.

Если вместо условий Неймана (2) заданы краевые условия Дирихле

$$u(0, \varepsilon) = u^0, \; u(1, \varepsilon) = u^1, \tag{5}$$

то при некоторых требованиях к u^0 и u^1 задача (1), (5) также имеет погранслойное решение с асимптотикой вида (4), но характер асимптотики изменяется. Так левый пограничный ряд строится теперь в виде

$$\Pi(\xi, \varepsilon) = \Pi_0(\xi, \varepsilon) + \sqrt{\varepsilon}\Pi_1(\xi, \varepsilon) + \varepsilon\Pi_2(\xi, \varepsilon) + \cdots, \; \xi = x/\varepsilon,$$

причем его коэффициенты зависят не только от ξ, но и от ε, в частности функция $\Pi_0(\xi, \varepsilon)$ является решением задачи (при условии, что $h = h(x)$):

$$\frac{d^2\Pi_0}{d\xi^2} = h(0)\left(\Pi_0^2 + 2\sqrt{\varepsilon}\overline{u}_1(0)\Pi_0\right), \; \xi > 0; \quad \Pi_0(0, \varepsilon) = u^0 - \varphi(0),$$

$$\Pi_0(\infty) = 0.$$

Анализ решения этой задачи и задач для следующих функций $\Pi_i(\xi, \varepsilon)$ показывает, что пограничный слой в задаче (1), (5) разделяется на три зоны. В первой зоне ($0 \le x \le \varepsilon^{1-\gamma}$, $0 < \gamma < 1/4$) функции $\Pi_i(\xi, \varepsilon)$ убывают с ростом ξ степенным образом: $\Pi_i(\xi, \varepsilon) = O\left(\frac{1}{(1+\xi)^2}\right)$, во второй зоне ($\varepsilon^{1-\gamma} \le x \le \varepsilon^{3/4}$) происходит изменение масштаба погранслойной переменной и характера убывания пограничных функций, и, наконец, в третьей зоне ($x \ge \varepsilon^{3/4}$) функции Π_i убывают экспоненциально, как $\exp(-\kappa\zeta)$, где $\zeta = \frac{x}{\varepsilon^{3/4}}$.

Если в левую часть уравнения (1) добавлен член с первой производной вида $\varepsilon A(x)u'$, то в случае кратного (в отличие от простого) корня вырожденного уравнения это приводит к изменению масштаба погранслойных переменных.

В докладе более подробно будет рассказано об упомянутых задачах, а также о некоторых задачах в случае кратного корня вырожденного уравнения для параболических и эллиптических уравнений. Для всех этих задач доказано существование решений с построенной погранслойной асимптотикой.

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О бифуркациях, повлекших удлинение ледниковых циклов в период плиоцена/плейстоцена

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В плиоцене (примерно за пять - два миллиона лет до настоящего времени) глобальный климат колебался с периодом, хорошо соответствовавшим 41-тысячелетнему изменению наклона оси вращения Земли к плоскости эклиптики. Затем этот период исчез несмотря на то, что 41-тысячелетний цикл даже немного увеличил свой размах, следовательно, климатический отклик на него должен был бы только усилиться. Анализируя палеоклиматические ряды, охватывающие плиоцен и последующий плейстоцен, мы показываем, что этот отклик просто стал неустойчивым и потому ненаблюдаемым. Одновременно посредством бифуркации удвоения периода, хорошо известной в теории нелинейных динамических систем, возбудились новые устойчивые, и потому наблюдаемые, климатические колебания. Позднее они испытали несколько вторичных бифуркаций, при которых их период поочередно утраивался и удваивался.

Об использовании теоремы Тихонова для вариационного описания равновесия в многостадийной математической модели транспортных потоков крупного мегаполиса

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Пусть в некотором городе имеется n районов и $N \gg n^2 \gg 1$ жителей, $L_i > 0$ - число жителей i-го района, $W_j > 0$ - число работающих в j-м районе. Обозначим через $d_{ij}(t) \geq 0$ - число жителей, живущих в i-м районе и работающих в j-м в момент времени t. Естественная эволюция жителей в медленном времени (годы) задается СОДУ:

$$\frac{d}{dt}c_{ij} = \sum_{k,p=1}^{n} \gamma \exp\left(\beta\left([T_{ip} + T_{kj}] - [T_{ij} + T_{kp}]\right)\right) c_{ip}c_{kj} -$$

$$-\sum_{k,p=1}^{n} \gamma \exp\left(\beta\left([T_{ij} + T_{kp}] - [T_{ip} + T_{kj}]\right)\right) c_{ij}c_{kp},$$

где $c_{ij} = d_{ij}/N$, $\beta, \gamma > 0$ - некоторые параметры, T_{ij} интерпретируются как затраты в пути из i-го района в j-й. Заметим, что $\sum\limits_{j=1}^{n} c_{ij}(t) = L_i/N \equiv l_i$, $\sum\limits_{i=1}^{n} c_{ij}(t) = W_j/N \equiv w_j$, $i, j = 1, \ldots, n$. Предложенная динамика при фиксированных T_{ij} имеет функцию Ляпунова: $H(c) = \sum\limits_{i,j=1}^{n} c_{ij}\ln(c_{ij}) + \beta \sum\limits_{i,j=1}^{n} c_{ij}T_{ij}$, и приводит к единственному ассимптотически глобально устойчивому стационару $d^* = c^*N$, который описывается классической энтропийной моделью расчета матрицы корреспонденций d.

Пусть транспортная сеть города представлена ориентированным графом $\Gamma = (V, E)$, где V - вершины графа, $|V| = n$; $E \subset V \times V$ - ребра графа. Пусть $W = \{w = (i,j) : i, j \in V\}$ - множество корреспонденций; P_w - множество путей, отвечающих корреспонденции $w \in W$; $P \equiv \bigcup_{w \in W} P_w$; x_p [автомобилей/час] - величина потока по пути p, $x = \{x_p : p \in P\}$; f_e– величина потока по дуге e:

$$f_e(x) = \sum_{p \in P} \delta_{ep} x_p, \text{ где } \delta_{ep} = \begin{cases} 1, & e \in p \\ 0, & e \notin p \end{cases};$$

$\tau_e(f_e) = \bar{t}_e \cdot \left(1 + \eta \cdot \left(f_e / \bar{f}_e\right)^{1/\mu}\right)$ – удельные затраты на проезд по дуге e, $1 \gg \mu > 0$, $\eta > 0$ - некоторые параметры, $\bar{t} = \{\bar{t}_e : \; e \in E\}$, $\bar{f} = \left\{\bar{f}_e : \; e \in E\right\}$; $G_p(x) = \sum\limits_{e \in E} \tau_e(f_e(x))\, \delta_{ep}$ - затраты на проезд по пути p.

В "жизни" T "подстраиваются" под d в быстром времени (дни). Этот процесс напоминает "нащупывание" равновесия Нэша в эволюционной игре с динамикой:

$$\varepsilon \frac{d}{dt} \tilde{x}_p = \frac{\tilde{x}_p \exp\left(-\alpha \tilde{G}_p(\tilde{x})\right)}{\sum\limits_{p' \in P_w} \tilde{x}_{p'} \exp\left(-\alpha \tilde{G}_{p'}(\tilde{x})\right)} - \tilde{x}_p,$$

где $\tilde{x}_p = x_p / d_w$, $\tilde{G}_p(\tilde{x}) = G_p(x)$, $p \in P_w$, $w \in W$, $1 \gg \varepsilon > 0$, $\alpha > 0$ - некоторые параметры. Эта игра потенциальная, и ее функция Ляпунова имеет вид: $\Psi(f_e(x)) = \sum\limits_{e \in E} \sigma_e(f_e(x))$, где $\sigma_e(f_e) = \int\limits_0^{f_e} \tau_e(z)\, dz$. Минимум строго выпуклой функции $\Psi(f_e(x))$ на компакте $X = \{x \geq 0 : \; \sum\limits_{p \in P_w} x_p = d_w, \; w \in W\}$ при фиксированных корреспонденциях d_w дает (единственное) ассимптотически глобально устойчивое равновесное распределение потоков по ребрам f, соответствующее классической модели Бэкмана с функциями затрат $\tau_e(f_e)$ типа BPR, зашитой в подавляющее большинство современных прикладных транспортных пакетов типа EMME/3, PTV.

Положим $t_e \equiv \tau_e(f_e)$. Обозначим через $T_{ij}(t)$ длину кратчайшего пути на графе Γ, ребра которого взвешены согласно t. Тогда если допустить, что описанные выше отдельно процедуры формирования матрицы корреспонденций d в медленном времени и равновесного распределения потоков по ребрам f в быстром времени сосуществуют, то в пределе Нестерова-деПальма $\mu \to 0+$ поиск глобально устойчивой (стационарной) равновесной конфигурации сводится (с помощью теоремы

Тихонова о разделении времени) к решению задачи:

$$\min_{t\geq\bar{t},\ \lambda^L,\ \lambda^W}\left\{\ln[\sum_{i,j=1}^{n}\exp(-\beta T_{ij}(t)-\lambda_i^L-\lambda_j^W)]+\sum_{i=1}^{n}\lambda_i^L l_i+\sum_{j=1}^{n}\lambda_j^W \mathrm{w}_j+\right.$$

$$\left.+\beta\langle\bar{f},t-\bar{t}\rangle\right\}, \qquad (*)$$

причем $d_{ij}=\tilde{Z}^{-1}\exp\left(-\beta T_{ij}(t)-\lambda_i^L-\lambda_j^W\right)$, где $\tilde{Z}^{-1}$ ищется из условия нормировки $\sum\limits_{i,j=1}^{n} d_{ij}=N$, $f=\bar{f}-s$, а s - (оптимальный) вектор двойственных множителей для ограничений $t\geq\bar{t}$ в задаче $(*)$. Таким образом, описание равновесной конфигурации свелось к решению задачи негладкой выпуклой оптимизации $(*)$ каким-нибудь прямо-двойственным методом, что эффективно можно делать на практике.

Детали см. http://dcam.mipt.ru/science/seminars/traffic_flow.html

Катастрофа голубого неба в релаксационных системах

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Феномен, получивший название "катастрофа голубого неба", был одним из многочисленных объектов теории динамических систем, который в свое время привлек внимание Л.П. Шильникова. Напомним, что речь идет о нелокальной бифуркации коразмерности один, состоящей в простейшем случае в следующем. Пусть гладкое однопараметрическое семейство векторных полей X_μ в $\mathbb{R}^3$ имеет при $\mu=0$ периодическую траекторию L_0 типа "простой седло-узел" и некоторая достаточно малая окрестность U этой траектории разделяется двумерным сильно устойчивым многообразием $W^{ss}(L_0)$ на узловую область U_+, все траектории из которой стремятся к L_0 при $t\to+\infty$, и седловую область U_-, содержащую двумерное неустойчивое многообразие $W^u_{loc}(L_0)$ с краем L_0. Пусть все траектории системы X_0 с начальными условиями из $W^u_{loc}(L_0)$ при увеличении t сначала покидают окрестность U, а затем снова возвращаются в нее, попадая в U_+. Пусть, наконец, продолжение неустойчивого многообразия $W^u_{loc}(L_0)$ по траекториям потока X_0, не является топологическим многообразием. Тогда, как показано в [1] при некоторых дополнительных условиях, исчезновение в системе X_μ, $0 < \mu \ll 1$ цикла L_0 приводит к появлению устойчивой замкнутой траектории $L(\mu)$ такой, что при $\mu \to 0$ ее период и длина стремятся к бесконечности. В [2] продемонстрирована реализуемость такой бифуркации в сингулярно возмущенных системах с одной медленной и m, $m \geq 2$, быстрыми переменными. Нами установлены [3] условия, при которых катастрофа голубого неба наблюдается в релаксационных системах с двумя медленными и одной быстрой переменной.

Литература

[1] Тураев Д. В., Шильников Л. П. // ДАН. 1995. Т. 342. № 5. С. 596–599.
[2] Shilnikov A., Shilnikov L., Turaev D. // Moscow Math. J. 2005. V. 5. № 1. P. 269–282.
[3] Глызин С.Д., Колесов А.Ю., Розов Н.Х. // Дифференц. уравнения. 2008. Т. 44.
№ 2. С. 158–171.

Гомоклинический каскад бифуркаций в экологической системе

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Рассматривается модель Розенцвейга-Макартура экологической системы типа "жертва-хищник-суперхищник", описываемая системой трёх дифференциальных уравнений с параметрами:

$$\zeta \dot{x} = x(1 - x - y/(\beta_1 + x)), \dot{y} = y(x/\beta_1 + x - \delta_1 - z/(\beta_2 + y)),$$

$$\dot{z} = \varepsilon z(y/(\beta_2 + y) - \delta_2). \qquad (1)$$

В качестве бифуркационных параметров рассматриваются ε и δ_2, параметры ζ, β_1, β_2, δ_1 фиксируются. Для особой точки $O(x^*, y^*, z^*)$, находящейся в области положительных x, y, z, построено разбиение плоскости параметров ε и δ_2 на области по типу грубой особой точки линеаризованной системы. При пересечении границы области седлофокуса с положительными действительными частями пары комплексно-сопряженных корней происходит бифуркация Андронова-Хопфа рождения устойчивого предельного цикла с последующим каскадом бифуркаций удвоения периода цикла и субгармоническим каскадом Шарковского, заканчивающегося рождением цикла периода три. При дальнейшем изменении параметров в системе появляются циклы гомоклинического каскада бифуркаций, приводящего к образованию странного аттрактора. С помощью преобразований системы и доказательных вычислений показано существование гомоклинической траектории седлофокуса, разрушение которой является главной бифуркацией гомоклинического каскада, и определена область параметров, в которой она существует. Получены бифуркационные диаграммы, графики показателей Ляпунова, графики седлового числа, фрактальные размерности странного аттрактора. Работа выполнена с применением системы аналитических и численных вычислений Maple-13.

Литература

[1] Шильников Л. П., Шильников А. Л., Тураев Д. В., Чуа Л. Методы качественной теории в нелинейной динамике. Часть 2. - М.-Ижевск: НИЦ "Регулярная и хаотическая динамика 2009, 548 с.

О бифуркациях соленоидальных инвариантных множеств

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В работе [1] был введен новый класс диффеоморфизмов с инвариантными соленоидальными множествами, который включает в себя классический пример Смейла диффеоморфизма полнотория в себя с одномерным растягивающимся аттрактором, являющимся соленоидом. Было доказано, что имеется два принципиально разных случая спектрального разложения неблуждающего множества. В докладе рассматриваются бифуркации перехода от одного случая к другому. Эти бифуркации можно рассматривать как бифуркации разрушения и восстановления растягивающихся аттракторов Смейла-Вильямса в классе специальных диффеоморфизмов.

Работа выполнена совместно с С.В. Гонченко.

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Литература

[1] Жужома Е.В., Исаенкова Н.В. О нульмерных соленоидальных базисных множествах // Матем. сборник. – 2011. – т. 202, N3. – С.47 – 68.

Признаки синхронизации субгармонических колебаний в консервативных динамических системах

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Явление возникновения в нелинейной динамической системе периодических колебаний с периодом, кратным периоду возмущающей силы, называют синхронизацией на субгармониках. Явление синхронизации может возникать как в диссипативных, так и в консервативных системах.

В докладе рассматривается динамическая система

$$x' = F(x; \mu; t), \quad x \in R^n, \tag{1}$$

где μ – скалярный параметр. Предполагаются выполненными следующие условия:

- функция $F(x; \mu; t)$ является непрерывно дифференцируемой по совокупности переменных и 2π-периодической по t;

- система (1) имеет точку равновесия $x = 0$ при всех μ, т.е. $F(0; \mu; t) \equiv 0$;

- функция $F(x; \mu; t)$ при $\mu = 0$ не зависит от t, при этом система $x' = G(x)$, где $G(x) = F(x; 0; t)$, является консервативной;

- матрица Якоби $G'_x(0)$ имеет собственные значения $\pm ri$, где r – рациональное число.

В докладе приводится схема исследования субгармонических колебаний уравнения (1), приводящая к новым признакам синхронизации на субгармониках. Рассмотрены некоторые приложения.

Статистическая необратимость динамических систем с мерой

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Эффект области в динамике распределенной кинетической системы

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Поведение распределенной кинетической системы в окрестности однородного состояния равновесия в плоской односвязной области K_μ с гладкой границей γ_μ описывает следующая начально-краевая задача

$$\partial u/\partial t = D(\varepsilon)\Delta u + A(\varepsilon)u + F(u;\varepsilon) \tag{1}$$

$$\partial u/\partial \nu \mid_{\gamma_\mu} = 0,\ u(x,y,0) = u_0(x,y), (x,y) \in K_\mu, t \geq 0, \tag{2}$$

где $u = u(x,y,t) \in R^n$ $((n \geq 3)$ – вектор, характеризующий величину отклонения концентраций веществ от равновесного состояния; ν – направление внешней нормали к границе K_μ ; $u_0(x,y)$ – начальная концентрация веществ; $0 < \varepsilon \leq \varepsilon_0$ – параметр; матрицы $D(\varepsilon) \equiv D^*(\varepsilon)$, $A(\varepsilon)$ порядка n и вектор-функция $F(u;\varepsilon) \in R^n$ $(F(0;\varepsilon) \equiv 0, \| F(.) \|_{R^n} = O(\| u \|^2_{R^n}))$ достаточно гладко зависят от входящих переменных,$D(\varepsilon)$ является положительно определенной и определяет коэффициенты диффузии веществ; Δ – оператор Лапласа. Предполагается, что область $K_\mu = \{(x,y) : x = \rho\cos(\phi), y = \rho\sin(\phi), 0 \leq \rho \leq R_0(1+\mu\delta(\phi)), 0 \leq \phi < 2\pi, 0 < \mu \leq \mu_0\}$, где $\delta(\phi)$ n – гладкая 2π-периодическая функция, K_0 – круг радиуса R_0.

Предположим, что при $\mu = 0$ решения начально-краевой задачи (1)-(2) из некоторой фиксированной окрестности нулевого решения стремятся к нулю при $t \to \infty$. Изучается возможность возникновения при $\mu > 0$ в задаче (1)-(2) устойчивых пространственно-неоднородных колебательных решений (аттракторов), в том числе хаотических, принадлежащих окрестности нулевого решения и обусловленных деформацией области K_0 в область K_μ. Сформулированы условия существования таких решений. В плоскости параметров (ε, μ) построены области существования пространственно неоднородных периодических решений, инвариантных торов, показана возможность существования серии бифуркаций удвоения периода, приводящая к хаотическому аттрактору. Такой механизм предлагается назвать эффектом области.

Кластеризация в моделях динамики пространственно распределенных популяций

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Для описания динамики реальных биологических популяций, представленных взаимодействующими локальными популяциями, обменивающиеся мигрантами, возможно использовать системы глобально связанных отображений. В данной работе изучаются закономерности формирования кластеров в зависимости от начального распределения фазовых переменных или начального распределения особей по ареалу, а так же, в зависимости от числа связей между элементами системы и формы ареала моделируемой популяции. Рассматривается два вида систем глобально связанных отображений с диссипативной и инерциальной связью.

Для изучения механизмов формирования кластеров предлагается оригинальная методика оценки близости пространственных распределений популяции по ареалу в различные моменты времени, который без необходимости определения бассейнов притяжения позволяет выделить области в параметрическом пространстве, в которых формируется та или иная фаза кластеризации. В качестве меры близости различных пространственных распределений использовался коэффициент детерминации, который вычисляется по двум или более наборам численностей локальных групп особей пространственно-распределенных популяций. В результате удалось показать, что увеличение числа связей, в том числе появление дальнодействующих, между локальными очагами, сопровождающееся усложнением формы ареала, увеличивает возможное число таких кластеров, с одновременным сужением области их существования в параметрическом пространстве. Показано, что при увеличении интенсивности миграционного взаимодействия уменьшается число возможных кластеров, а рост репродуктивных параметров не всегда приводит к усложнению динамических режимов.

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Ансамбль глобально связанных периодических осцилляторов с шумом, демонстрирующий гиперболический хаос

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Задачи о синхронизации в ансамблях осцилляторов традиционно вызывают большой интерес. В частности, когда количество осцилляторов стремится к бесконечности (термодинамический предел), большое значение приобретает вопрос о соотношении между характером микроскопической динамики и наблюдаемым поведением макроскопических параметров. В классическом случае микроскопическая динамика является хаотической, тогда как на макроуровне нерегулярность исчезает. Тем не менее известны обратные примеры, когда связанные периодические осцилляторы порождают хаотическую коллективную динамику. Один из таких случаев представлен в работе [1]. Эта, в некотором смысле контр-интуитивная, ситуация представляется весьма интересной. Однако изучение поведения в термодинамическом пределе ансамблей детерминированных осцилляторов сопряжено с большими сложностями из-за сильных искажений, обусловленных конечными размерами модельных систем. Чтобы обойти это затруднение, в настоящей работе рассматривается ансамбль стохастических осцилляторов. Исходный детерминированный осциллятор является периодическим, а добавляемый к нему шум достаточно мал, так что устойчивость индивидуальных траекторий сохраняется. Динамику этого ансамбля в термодинамическом пределе можно изучать с привлечением математического аппарата на основе нелинейного уравнения Фоккера–Планка.

Ансамбль строится таким образом, чтобы на макроуровне получить однородно гиперболический хаос. В основе лежит идея, подробно описанная в книге [2]. При её реализации для системы двух связанных осцилляторов, существенными компонентами являются периодическое внешнее воздействие, которое поочерёдно выводит подсистемы за порог генерации автоколебаний и нелинейная связь, обеспечивающая передачу возбуждения от одной подсистемы к другой, сопровождающуюся мультипликацией фазы возбуждения.

Мы рассматриваем ансамбль фазовых стохастических осцилляторов, которые взаимодействуют друг с другом через два поочерёдно включаемых средних поля, соответствующих первому и второму моментам распределения фаз ансамбля. Поле первого порядка вызывает синфазную синхронизацию осцилляторов, а при включении вместо него поля второго порядка при синхронизации осцилляторы распадаются на две группы, колеблющиеся в противофазе. Дополнительно на каждый осциллятор действует малая периодическая внешняя сила.

Анализ построенного стохастического ансамбля показал, что как и ожидалось, коллективная динамика в термодинамическом пределе является хаотической, на что указывает положительный показатель Ляпунова у уравнения Фоккера–Планка. Кроме того, численная проверка подтвердила, что хаос имеет гиперболическую природу. При этом на микроскопическом уровне рассмотрения хаос отсутствует, так как парциальные осцилляторы не имеют положительных показателей Ляпунова.

Список литературы

1. Kuznetsov S. P., Pikovsky A., Rosenblum M. Collective phase chaos in the dynamics of interacting oscillator ensembles // CHAOS. 2010. — DEC. T. 20, № 4.

2. Kuznetsov S. P. Hyperbolic Chaos: A Physicist’s View. Higher Education Press: Bijing and Springer-Verlag: Berlin, Heidelberg, 2012. C. 336.

Синхронизация в системе миграционно-связанных сообществ "ресурс–потребитель"

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С начала 20 века активно развивается динамическая теория биологических популяций, в рамках которой исследуется закономерности изменения численности популяций взаимодействующих биологических видов, в частности, условия возникновения устойчивых колебаний. Для описания динамики таких популяций необходимо использовать нелинейные модели, учитывающие основные факторы их развития (рождаемость, смертность, межвидовые взаимодействия, миграция).

Особый интерес представляет выявление условий синхронизации изменений численностей двух или многих сообществ, заселяющих соседние регионы и взаимодействующих между собой за счет локальной миграции особей.

Проведено исследование математической модели динамики численности в системе двувидовых сообществ типа "ресурс-потребитель связанных миграциями потребителя. Выполнено качественное описание поведения модели, определены условия синхронизации колебаний рассматриваемых сообществ, изучено влияние миграционного взаимодействия между сообществами на динамику каждой популяции.

В результате исследования показано, что введение коэффициента миграции в классическую модель типа "ресурс-потребитель приводит к синхронизации колебаний рассматриваемых систем. При этом синхронизируются как длина периода колебаний, так и амплитуда и фаза. Величина коэффициента миграции напрямую влияет на скорость синхронизации. Показано, что при малой интенсивности миграций для достижения полной синхронизации каждому сообществу может потребоваться разное число популяционных циклов, что связано с первоначальным различием в длинах периодов колебаний изолированных систем.

Исследование выполнено при финансовой поддержке ДВО РАН (конкурсные интеграционные проекты с СО РАН 12-II-СО-06-019, конкурсный проект 12-I-П28-02) и РФФИ (региональный проект 11-01-98512-р_восток_а).

Потоки Морса-Смейла с тремя неблуждающими точками на замкнутых n-мерных многообразиях

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Изучается топологическая классификация гладких потоков Морса-Смейла на замкнутых гладких n-мерных многообразиях M^n, у которых неблуждающее множе-ство Ω состоит ровно из k точек.

Для $k = 1$ потоков Морса-Смейла на замкнутых многообразиях не существует. Для $k = 2$, когда в неблуждающем множестве Ω только две точки, гладкие потоки Морса-Смейла проклассифицированы. Все такие потоки топологически эквивалент-ны. Они могут быть только на n-мерной сфере $S^n, n > 0$, и будут состоять из источника, стока и блуждающих траекторий, идущих от источника к стоку. Для $k = 3$ рассматриваемая задача решена частично в работе [1], в которой дана класси-фикация гладких потоков на замкнутых n-мерных многообразиях в случае, когда размерность многообразий $n < 5$.

Доклад посвящен описанию топологической структуры гладких потоков Морса- Смейла, заданных на гладких замкнутых n-мерных многообразиях $M^n, n > 1$, с неблуждающим множеством Ω, состоящем из трех точек.

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Литература

[1] Жужома Е.В., Медведев В.С. Системы Морса-Смейла с тремя неблуждающими точками // Доклады РАН – 2011.– Том 440, No. 1.– с. 11-14.

Режимы динамики двухкомпонентной популяционной модели, включающей промысловое изъятие

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В данной работе продолжено исследование дискретной математической модели, описывающей динамику численности двухвозрастной популяции, с плотностным лимитированием рождаемости. Коэффициент рождаемости выбран по аналогии с моделью Рикера и является функцией численности обеих возрастных групп. Такая формализация позволяет учесть плотностную регуляцию, приводящую к дифференцированному уменьшению интенсивности воспроизводства потомства с ростом численности различных возрастных групп. Предполагается, что популяция подвергается промысловому изъятию после сезона размножения, и количество изъятых особей пропорционально общей численности популяции (изымаются крупные половозрелые особи – старший возрастной класс).

Проведено подробное аналитическое и численное исследование предложенной модели. В зависимости от интенсивности процессов внутривидовой конкуренции и промыслового изъятия сделаны выводы о способах потери устойчивости нетривиального равновесия. Показано, что при лимитировании рождаемости численностью старшего возрастного класса увеличение объемов промыслового изъятия приводит к расширению области устойчивости нетривиального равновесия. Потеря устойчивости происходит по сценарию Неймарка-Сакера. При снижении рождаемости, в связи с ростом численности молоди, воздействие промыслового изъятия ведет к двухгодичным колебания численности. Как правило, рост интенсивности промыслового изъятия, приводит к стабилизации численности популяции (расширение области устойчивости). Однако в ряде численных экспериментов удалось выявить немонотонные изменения области устойчивости нетривиального равновесия, вызванные учетом возрастной структуры популяции.

Исследование выполнено при финансовой поддержке ДВО РАН (проекты 12-I-П30-14, 12-II-СУ-06-007, КПФИ 12-06-017) и РФФИ (проект 11-01-98512-р_восток_а).

Бифуркации и нелинейные эффекты, интерпретируемые в моделях воспроизводства популяций рыб

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Обсуждаются модели описания эффективности воспроизводства популяций рыб в рамках теории формирования пополнения с учетом нелинейных эффектов. Модели анализируются как непрерывно-дискретные динамические системы. Описываются взаимоисключающие выводы, возникающие при попытках биологической интерпретаций динамики двух применявшихся в ихтиологии дискретных итераций с бифуркациями удвоения периода. С использованием формализма гибридного автомата разработан метод исследования влияния изменений эколого-физиологических стадий развития молоди на характер воспроизводства. Существует концепция о зависимости между запасом и пополнением. Зависимость $R = aSe^{-bS}$ описывает снижение численности пополнения при увеличении запаса, когда повышенная плотность становится негативным фактором, увеличивающим смертность. Созданы модели на основе данных об искусственном и естественном воспроизводстве осетровых Каспия для выяснения диапазона их устойчивости к интенсивному промыслу. На основе формализации эффекта резкого снижения эффективности воспроизводства при деградации и влияние роста особей на темп убыли численности поколения:

$$\frac{dN}{dt} = -(\alpha w(t)N(t) + \theta(S)\beta)\,N(t),\ \frac{dw}{dt} = \frac{g}{N^k + \zeta},\ \theta(S) = \frac{1}{1 - \exp(-cS)}, \tag{1}$$

где S – величина запаса; $w(t)$ – уровень размерного развития поколения; g – параметр количества доступных кормовых объектов; $\theta(S) \to 1$ при $S \to \infty$, для учета эффекта резкого снижения эффективности воспроизводства при деградации нерестового стада; ζ – ограничение темпов развития, не зависящие от численности; $k \in [\frac{1}{2}, 1)$; $c < 1$ - степень выраженности эффекта Олли; α – коэффициент компенсационной смертности; β – декомпенсационной смертности; $t \in [0, T]$ – специфичный интервал уязвимости.

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Бифуркации в прикладных диссипативных динамических системах

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Изучается класс прикладных динамических систем вида

$$\dot{x} = F(x_1, x_2, ..., x_n; \mu_1, \mu_2, ..., \mu_k), \quad i = 1..n, \tag{1}$$

удовлетворяющих условию

$$\frac{\partial F_1}{\partial x_1} + \frac{\partial F_2}{\partial x_2} + ... + \frac{\partial F_n}{\partial x_n} < 0, \tag{2}$$

где x_i – фазовые переменные, а μ_i – параметры.

Классической примером такой системы является динамическая система Лоренца. В качестве следующего примера рассматривается система Лэнгфорда, полученная усечением бесконечной системы дифференциальных уравнений, первоначально введенной Хопфом в качестве возможной модели турбулентного течения несжимаемой (вязкой) жидкости. Установлено, что в системе Лэнгфорда реализуется бифуркация рождения двумерного тора. Далее рассматривается система Франческини-Тибальди, полученная путём пяти-модового усечения Фурье-разложения решения уравнений Навье-Стокса, описывающих течение вязкой жидкости. Установлено, что в этой системе имеет место бифуркация Андронова-Хопфа при прохождении числа Рейнольдса r через критическое значение $r = r_{kr} = 22.8537016318316527$, а при $r = 28.8$ рождается хаотический аттрактор Франческини-Тибальди. Однако, при $r = 34$ нерегулярный режим в этой системе сменяется автоколебательным режимом и он сохраняется при дальнейшем увеличении параметра r.

Также в докладе рассматриваются свойства решений систем вида (1), (2) на базе динамических систем Вышнеградского-Понтрягина, Анищенко-Астахова, Чумакова-Слинько и Рикитаке.

Явления взрыва решений дифференциальных уравнений

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Явление взрыва решения предлагается определить как разрыв многозначного отображения, сопоставляющего начально-краевой задаче множество решений этой задачи. Будет показано, что такое определение охватывает как эффект разрушения, так и эффект неединственности решения и задает процедуру регуляризации некорректных задач.

Рассмотрим задачу Коши для дифференциального уравнения как уравнение

$$\mathbf{A}u = f, \quad f \in X, \quad u \in Y, \quad \mathbf{A} \in B(Y, X), \tag{1}$$

где X, Y – банаховы пространства, а $B(Y, X)$ – некоторое банахово пространство операторов, действующих из области определения $D(\mathbf{A}) \subset Y$ в пространство X.

Задача Коши (1) определяет многозначное отображение

$$G: \; X \times B(Y, X) \to 2^Y,$$

заданное на множестве $X \times B(Y, X)$ и принимающее значение во множестве 2^Y всех подмножеств пространства Y, определяемое формулой

$$G(f, \mathbf{A}) = M_{f,\mathbf{A}} \equiv \mathbf{A}^{-1}(f).$$

Заметим, что отображение G определено в любой точке пространства $X \times B(Y, X)$, в некоторых точках которого значением отображения G может быть пустое можество.

Рассмотрим отображение G как отображение банахова пространства $X \times B(Y, X)$ в топологическое пространство 2^Y.

Определение. Будем говорить, что задача Коши (1) проявляет свойство взрыва, если точка $(f, \mathbf{A})$ является точкой разрыва отображения G.

Будут рассмотрены примеры явлений взрыва решений обыкновенных дифференциальных уравнений и уравнений с частными производными. Будут исследованы предельные переходы в пространстве мер, ассоциированных с решениями задачи Коши (1).

Возмущения сингулярно гиперболических аттракторов

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Определение сингулярно гиперболического потока (или сингулярно гиперболического множества) было приведено в [1] как обобщение гиперболического потока на случай, когда в множестве неблуждающих точек содержится неподвижная точка. Похожее определение приведено в [2], где соответствующий поток назван псевдогиперболическим. Приведем определение сингулярно гиперболического потока (модификация определения из работы [1]).

Определение. *Поток Φ_t на римановом многообразии M размерности n, порожденный векторным полем $X(x)$, называется сингулярно гиперболическим на инвариантном множестве Λ, если касательное пространство T_xM в каждой точке $x \in \Lambda$ раскладывается в прямую сумму двух инвариантных пространств $T_xM = E_x^{ss} \oplus E_x^c$, непрерывно зависящих от x на Λ, причем выполняются свойства.*

1. E_x^{ss} $(n-1)$-мерно, E_x^c двумерно.

2. Существуют константы $c_1 > 0$, $c_2 > 0$, $\gamma_1 > 0$, $\gamma_2 > 0$ такие, что

(a) Если $u \in E_x^{ss}$, $t > 0$, то $|d\Phi_t(u)| < c_1 e^{-\gamma_1 t}|u|$;

(b) Если $u \in E_x^{ss}$, $v \in E_x^c$, $t > 0$, то $\dfrac{|d\Phi_t(u)|}{|u|} < c_2 e^{-\gamma_2 t}\dfrac{|d\Phi_t(v)|}{|v|}$.

3. Существуют константы $c_3 > 0$, $\gamma_3 > 0$ такие, что если $u, v \in E_x^c$, $t > 0$, а $S(u,v)$ обозначает площадь параллелограмма, порожденного векторами u, v, то при всех $t > 0$ верно неравенство

$$S(d\Phi_t(u), d\Phi_t(v)) > c_3 e^{\gamma_3 t} S(u,v).$$

4. Все неподвижные точки, лежащие в множестве Λ, гиперболические.

Предполагается, что существует такое открытое множество $U \subset M$, что $\Lambda = \cap_{t>0}\Phi_t(U)$.

В работе [3] доказано, что существует конечное число замкнутых подмножеств $\Lambda_1, \ldots, \Lambda_k \subset \Lambda$ (эти множества называются эргодическими компонентами) и инвариантных мер $\mu_1, \ldots, \mu_k$, сосредоточенных на множествах Λ_j , такие, что на множествах Λ_j существует инвариантное семейство строго неустойчивых многообразий, определенных на множестве полной меры, поток Φ_t на каждом множестве Λ_j с мерой μ_j эргодичен. Кроме того, периодические траектории плотны в множествах Λ_j, и если мера ν абсолютно непрерывна относительно меры Лебега на U, то семейство мер

$$\nu_T = \frac{1}{T}\int_0^T \Phi_t(\nu)\, dt$$

при $T \to \infty$ сходится к мере $\sum p_j \mu_j$ с некоторыми коэфициентами $p_j \geq 0$, $\sum p_j = 1$ (эта мера называется мерой Синая-Боуэна-Рюэлля, или SBR-мерой).

Если множество Λ_j содержит неподвижную точку, то возможно два случая: либо существует собственная функция, либо поток на Λ_j с мерой μ_j является перемешивающим.

Многие свойства сингулярно гиперболических потоков сохраняются при возмущениях. В частности, само свойство сингулярной гиперболичности при возмущении сохраняется.

Если последовательность векторных полей $X_n(x)$ сходится в C^1-топологии к векторному полю $X(x)$, причем все векторные поля $X_n(x)$ равномерно ограничены в C^2-топологии, μ_n — последовательность SBR-мер для $X_n(x)$, μ — предельная мера последовательности μ_n, то μ — SBR-мера для $X(x)$.

Число эргодических компонент при малом возмущении не увеличивается.

Литература

[1] C. Morales, M.J. Pacifico, E. Pujals, “On C^1 robust singular transitive sets for three dimensional flows,” *C. R. Acad. Sci. Paris*, 326, No. 22, 81–86 (1998).

[2] Тураев Д. В., Шильников Л.П., “Пример дикого странного аттрактора”, *Матем. сб.*, 189, No. 2, 137–160 (1998).

[3] Сатаев Е.А., “Инвариантные меры для сингулярно гиперболических аттракторов”, *Матем. сб.*, 201, No 3, 107—160 (2010).

КАМ-торы около резонанса

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Изучаются квазипериодические движения гамильтоновых систем в резонансных зонах и проекции соответствующих КАМ-торов на пространство переменных "действие".

Бифуркации и странные аттракторы в моделях динамики и эволюции биологических популяций

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Данное сообщение посвящено анализу роли математического моделирования при формировании и развитии базовых концепций общей биологии: теории естественного отбора, синтетической теории эволюции, теории экологического регулирования популяционной динамики. Обсуждаются некоторые подходы к синтезу популяционно-генетических и экологических идей, а также результаты анализа и моделирования популяционной динамики некоторых конкретных видов животных, обитающих в Дальневосточном регионе. Приводятся результаты математического моделирования, объясняющие эволюционные закономерности возникновения сложных колебательных режимов динамики численности лимитированных популяций.

Проведен модельный анализ связи между продолжительностью онтогенеза и характером динамического поведения биологического сообщества. Показано, что в процессе естественной эволюции природной популяции с выраженной сезонностью жизненного цикла происходит закономерный переход от устойчивых режимов динамики численности к колебаниям и хаосу (псевдостохастическому поведению). Для более сложных нелинейных моделей динамики популяций с возрастной структурой (продолжительным онтогенезом) увеличение средней индивидуальной приспособленности приводит к возникновению хаотических аттракторов. Увеличение продолжительности и сложности онтогенеза "в среднем"увеличивает степень хаотизации аттракторов. Вместе с тем, выявлены резонансные значения репродуктивного потенциала при продолжительном онтогенезе, которые обеспечивают окна регуляризации в хаотической динамике. Можно сказать, что удлинение и усложнение онтогенеза, увеличивая "в среднем"хаотизацию, в конечном итоге способно обеспечить переход "от хаоса к порядку"и даже привести к устойчивым динамическим режимам.

Исследование поддержано ДВО РАН (конкурсные проекты 12-I-П28-02, 12-I-П30-14, 12-I-ОБН-05) и РФФИ (региональный проект 11-01-98512-р_восток_а).

Уравнение Рикера с циклически изменяющимся параметром

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Многие биологические виды имеют периодические жизненные циклы. Среди них выделяются такие виды, у которых точно определен период размножения и смежные поколения не перекрываются. Если предположить, что условия среды обитания этих видов имеют периодический характер изменений от поколения к поколению и численность последующего поколения зависит от численности предыдущего поколения, то в этом случае динамику численности одновозрастной популяции можно описать в рамках детерминистической модели Рикера с периодическим параметром. Этот параметр, также как и в классической модели, характеризует «емкость» экологической ниши и репродуктивный потенциал популяции. Период изменений значений параметра выбирается в соответствии с длиной периода учитываемых изменений в среде обитания. В данной работе рассмотрена модель Рикера с периодическим параметром, имеющем период длины 2.

Были построены карты периодических режимов, которые позволили заключить следующее. В рассмотренной области пространства параметров обнаруживаются устойчивые (по Ляпунову) циклы четной длины. Область фазового пространства системы поделена на бассейны притяжений сосуществующими аттракторами. При этом имеет место деление двух типов. Односвязный бассейн притяжений одного периодического решения своим расположением разбивает бассейн притяжения второго решения на две подобласти (1-ый тип). Деление фазового пространства бассейнами притяжения с фрактальной структурой. Каждый из бассейнов представляет собой многосвязное множество. Подобласти бассейнов чередуются в фазовом пространстве, напоминая фрактальную структуру (2-ой тип).

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