About laws of conservation in the dynamics of fluid

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Classical quantum-hydrodynamic analogy

(E. Madelung, 1929)

$$ih\psi_t = -\frac{h^2}{2m}\Delta\psi + V\psi, \qquad (1)$$

$$h = 1.054 \times 10^{-27} \text{ erg s}$$

$$\psi = \sqrt{\rho}\exp\left(i\frac{S}{h}\right)$$

$$\begin{cases} \rho_t + \operatorname{div}(\rho\nabla S) = 0 \\ S_t + \frac{1}{2}(\nabla S)^2 + V - \frac{h^2}{2}\frac{\Delta\left(\sqrt{\rho}\right)}{\sqrt{\rho}} = 0 \end{cases} \qquad (2)$$

The Lax representation of hydrodynamics equations

$$\boldsymbol{v}_t + (\boldsymbol{v}\nabla)\boldsymbol{v} = -\nabla P, \qquad \rho_t + \mathrm{div}(\rho\boldsymbol{v}) = 0,$$

$$P = \int \frac{dp}{\rho(p)}.$$

$$\left(\frac{\mathrm{rot}\boldsymbol{v}}{\rho}\right)_t = \left(\frac{\mathrm{rot}\boldsymbol{v}}{\rho}\cdot\nabla\right)\boldsymbol{v} - (\boldsymbol{v}\cdot\nabla)\left(\frac{\mathrm{rot}\boldsymbol{v}}{\rho}\right), \quad (3)$$

$$\hat{L}_t = i\left(\hat{L}\hat{A} - \hat{A}\hat{L}\right), \quad (4)$$

$$\hat{L} = -i\left(\frac{\mathrm{rot}\boldsymbol{v}}{\rho}\cdot\nabla\right), \qquad \hat{A} = -i(\boldsymbol{v}\cdot\nabla). \quad (5)$$

New quantum-hydrodynamic analogy

$$i\psi_t = \hat{A}\psi, \qquad \psi_t + (\boldsymbol{v} \cdot \nabla)\psi = 0, \qquad (6)$$

$$\frac{d}{dt}\hat{L} = \hat{L}_t + i\left(\hat{A}\hat{L} - \hat{L}\hat{A}\right) = 0,$$

$$\frac{d}{dt}\int_{-\infty}^{+\infty} \psi^* \hat{L}\psi d^3\,\boldsymbol{x} = 0.$$

New laws of conservation

If ψ satisfies the Schrodinger equation and $\hat{L}$ is the conserved operator, $\hat{L}\psi$ satisfies the Schrodinger equation. (Ertel's relation).

$$\rho(x, y, z, t)\frac{K}{\omega} = \mathrm{const}, \qquad (7)$$

$K = \dfrac{dl}{dl_0}$ is the vortex – line tension coefficient.

$$\left(\frac{\mathrm{rot}\boldsymbol{v}}{\rho} \cdot \nabla\right)^n \frac{K\rho}{\omega} = \mathrm{const}, \qquad \mathrm{n} = \overline{0, \infty}. \quad (8)$$

Average of operator is constant value if the operator conserved.

In case of incompressible inviscid fluid:

$$\int_{-\infty}^{+\infty} \psi^* \hat{L}^n \psi d^3\boldsymbol{x} = \mathrm{const}, n = \overline{0, \infty}. \qquad (9)$$

In case of barotropic motions of a compressible inviscid fluid:

$$\int_{-\infty}^{+\infty} \psi^* \left(\frac{\mathrm{rot}\boldsymbol{v}}{\rho} \cdot \nabla\right)^n \psi \rho d^3\boldsymbol{x} = \mathrm{const}, n = \overline{0, \infty}. \qquad (10)$$

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