International Conference-School
“Dynamics, Bifurcations and Chaos 2015 (DBC II)”
Nizhny Novgorod, July 20-24, 2015

Feynmanons in gravitational waves in fluid

***A. A. Potapov* — Dr. Sc. (Phys.-Math.), Professor, Institute of Radio Engineering and Electronics, Russian Academy of Sciences (IREE RAS), Moscow**

***A. E. Rassadin* — co-ordinator of united scientific and educational programmes of Nizhny Novgorod regional division of A. S. Popov’s Scientific and Technical Society of Radio engineering, Electronics and Communication**

Linear gravitational waves in incompressible infinitely deep fluid

$$\Delta_{\perp}\varphi + \frac{\partial^2 \varphi}{\partial z^2} = 0 \quad \Delta_{\perp} = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2}$$

— transversal Laplacian

$$\vec{x} = (x_1, x_2)$$

Potential of velocity of fluid:

$$\varphi(t, \vec{x}; z) = A \cdot \exp(|\vec{p}| \cdot z) \cdot \exp(i \cdot \vec{p} \cdot \vec{x} - i \cdot \omega \cdot t)$$

Dynamic boundary condition:

$$\vec{p} = (p_1, p_2)$$

$$\left[\frac{\partial \varphi}{\partial z} + \frac{1}{g} \cdot \frac{\partial^2 \varphi}{\partial t^2} \right]_{z=0} = 0$$

Dispersion relation:

$$\omega^2 = g \cdot |\vec{p}|$$

$$\Phi(t,\vec{x}) \equiv \varphi(t,\vec{x};0)$$ — **surface potential**

$$\Phi(t,\vec{x}) = A\cdot\exp(i\cdot\vec{p}\cdot\vec{x} - i\cdot\sqrt{g\cdot|\vec{p}|}\cdot t) + B\cdot\exp(i\cdot\vec{p}\cdot\vec{x} + i\cdot\sqrt{g\cdot|\vec{p}|}\cdot t)$$

Motivation: surface of fluid is

$$\zeta(t,\vec{x}) = -\frac{1}{g}\cdot\left.\frac{\partial\varphi(t,\vec{x};z)}{\partial t}\right|_{z=0}$$

$$\vec{p} \Rightarrow -i\cdot\nabla_{\perp}$$

Restoration of the equation for surface potential:

$$\frac{\partial^2\Phi}{\partial t^2} + g\cdot\sqrt{-\Delta_{\perp}}\cdot\Phi = 0$$

Rewrite the equation of the second order as the system of the first order:

$$\frac{\partial\Phi_1}{\partial t} = -\sqrt{g}\cdot(-\Delta_{\perp})^{\frac{1}{4}}\cdot\Phi_2 \qquad \frac{\partial\Phi_2}{\partial t} = \sqrt{g}\cdot(-\Delta_{\perp})^{\frac{1}{4}}\cdot\Phi_1$$

How to deal with fractional Laplacians? Very simple!

$$f(\vec{x}) = \int \tilde{f}(\vec{p}) \cdot \exp(i \cdot \vec{p} \cdot \vec{x}) \cdot \frac{d^3 p}{(2 \cdot \pi)^3}$$

$$(-\Delta)^{\alpha} f(\vec{x}) = \int | \vec{p} |^{2 \cdot \alpha} \cdot \tilde{f}(\vec{p}) \cdot \exp(i \cdot \vec{p} \cdot \vec{x}) \cdot \frac{d^3 p}{(2 \cdot \pi)^3}$$

$$\sqrt{-\Delta_{\perp}} \cdot f(\vec{x}) = \frac{1}{2 \cdot \pi} \cdot \int \frac{f(\vec{x}) - f(\vec{x}')}{| \vec{x} - \vec{x}' |^3} \cdot d^2 x'$$

$$(-\Delta_{\perp})^{\frac{1}{4}} f(\vec{x}) = \frac{\Gamma^2(1/4)}{16 \cdot \pi^2} \cdot \int \frac{f(\vec{x}) - f(\vec{x}')}{| \vec{x} - \vec{x}' |^{5/2}} \cdot d^2 x'$$

Uchaikin V. V., 2008

$$(-\Delta_{\perp})^{-\frac{1}{4}} f(\vec{x}) = \frac{1}{\Gamma^2(1/4)} \cdot \int \frac{f(\vec{x}') \cdot d^2 x'}{| \vec{x} - \vec{x}' |^{3/2}}$$

Rewrite our system as Schrödinger type equation:

$$i \cdot \frac{\partial |\Phi>}{\partial t} = \hat{H} \cdot |\Phi> \qquad \hat{H} = \sqrt{g} \cdot (-\Delta_\perp)^{\frac{1}{4}} \cdot \sigma_y \qquad |\Phi> = \begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix}$$

$$\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$ — **Pauli matrix** $$\sigma_y^2 = 1$$

$$|\Phi(t)> = \exp[-i \cdot t \cdot \sqrt{g} \cdot (-\Delta_\perp)^{\frac{1}{4}} \cdot \sigma_y] \cdot |\Phi(0)>$$

$$\Phi(t, \vec{x}) = \int \Gamma(\vec{x}, \vec{x}'; t) \cdot \Phi(0, \vec{x}') \cdot d^2 x'$$

$$\Gamma(\vec{x}, \vec{x}'; t) = <\vec{x} | \exp[-i \cdot t \cdot \sqrt{g} \cdot (-\Delta_\perp)^{\frac{1}{4}} \cdot \sigma_y] | \vec{x}'>$$ — **Green's matrix**

$$\exp[-i\cdot t\cdot\sqrt{g}\cdot(-\Delta_{\perp})^{\frac{1}{4}}\cdot\sigma_y]=\frac{1-\sigma_y}{2}\cdot\exp[i\cdot t\cdot\sqrt{g}\cdot(-\Delta_{\perp})^{\frac{1}{4}}]+\frac{1+\sigma_y}{2}\cdot\exp[-i\cdot t\cdot\sqrt{g}\cdot(-\Delta_{\perp})^{\frac{1}{4}}]$$

$$\Gamma(\vec{x},\vec{x}';t)=\frac{1-\sigma_y}{2}\cdot G^*(\vec{x}',\vec{x};t)+\frac{1+\sigma_y}{2}\cdot G(\vec{x},\vec{x}';t)$$

Green's function:

$$G(\vec{x},\vec{x}';t)=<\vec{x}\,|\exp[-i\cdot t\cdot\sqrt{g}\cdot(-\Delta_{\perp})^{\frac{1}{4}}]|\,\vec{x}'>$$

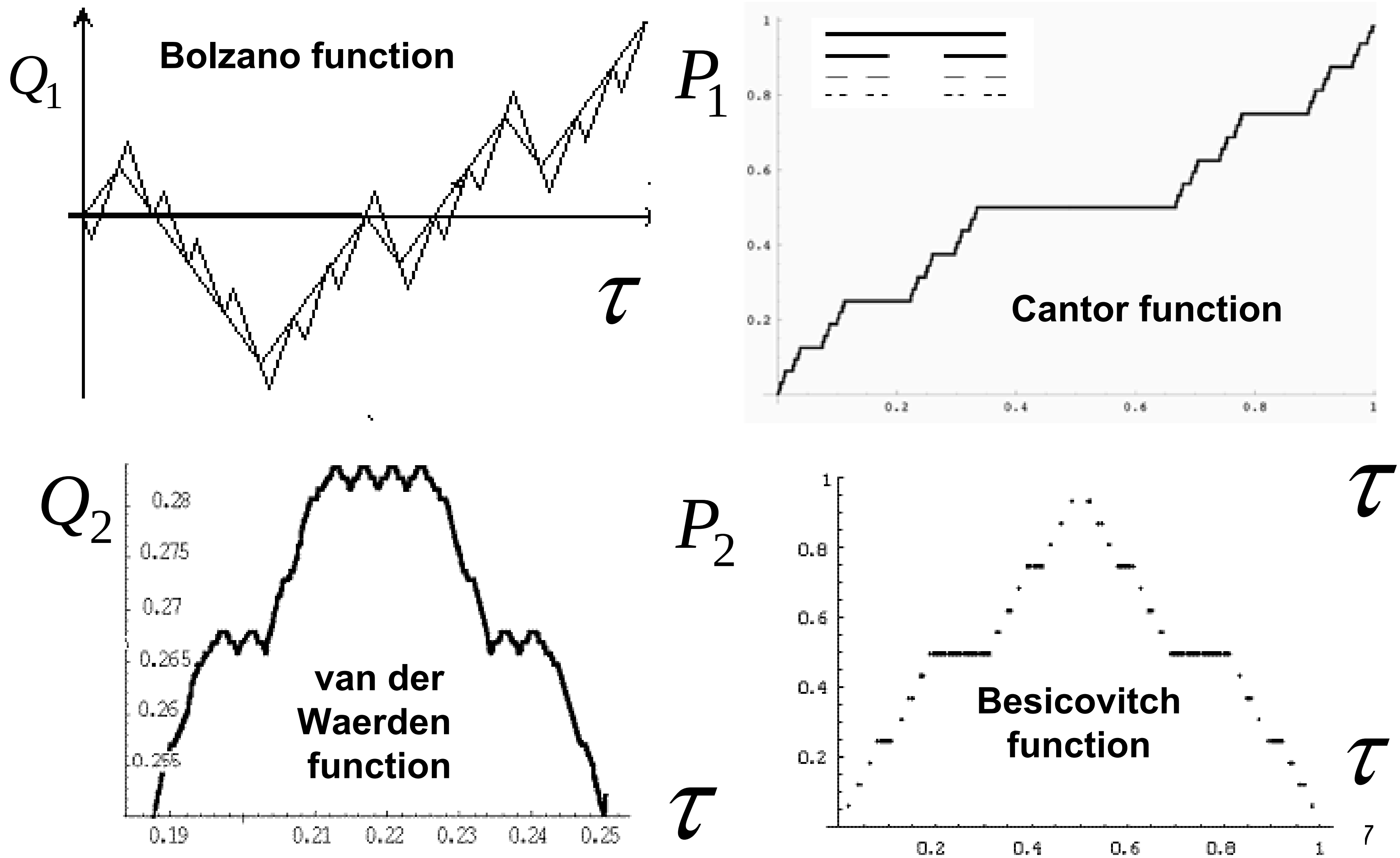
Green's function as Feynman integral:

$$G(\vec{x},\vec{x}';t)=\int_{\vec{Q}(0)=\vec{x}'}^{\vec{Q}(t)=\vec{x}}\exp[i\cdot\int_0^t(\vec{P}(\tau)\cdot\dot{\vec{Q}}(\tau)-\sqrt{g\cdot|\vec{P}(\tau)|})\cdot d\tau]\cdot\prod_{\tau}\frac{d\vec{P}(\tau)\cdot d\vec{Q}(\tau)}{2\cdot\pi}$$

Green's function as Fourier integral:

$$G(\vec{x},\vec{x}';t)=\int\exp[i\cdot\vec{p}\cdot(\vec{x}-\vec{x}')-i\cdot\sqrt{g\cdot|\vec{p}|}\cdot t]\cdot\frac{d^2p}{(2\cdot\pi)^2}$$

Fractal properties of trajectories in 4D phase space (P,Q): for coordinates (from the left) and for momenta (from the right)



Representation of Green's function by Feynman integral gives us the possibility to introduce quantum quasiparticle as object moving along nondifferentiable trajectories in four-dimensional phase space (P,Q). We call this quasiparticle by "surface hydron" (special case of "feynmanon")

$$\vec{Q}(\tau) = \vec{Q}_j + (\vec{Q}_{j+1} - \vec{Q}_j)\cdot(\tau - \tau_j)/\Delta\tau, \quad \vec{P}(\tau) = \vec{P}_{j+1}, \quad \tau \in [\tau_j, \tau_{j+1}]$$

$$\Delta\tau = \frac{t}{N+1} \qquad \tau_j = j\cdot\Delta\tau$$

$$j = \overline{0, N} \qquad N \to \infty$$

Four-dimensional random walk on the lattice with decreasing step is subset of phase trajectories of surface hydron.

"Instantaneous" dynamics of momenta may obey to succession maps:

$$\vec{P}_{j+1} = \vec{f}_1(\vec{P}_j), \quad \vec{P}_{j+2} = \vec{f}_2(\vec{P}_j, \vec{P}_{j+1}),$$

$$\ldots,$$

$$\vec{P}_{j+l} = \vec{f}_l(\vec{P}_j, \vec{P}_{j+1}, \ldots, \vec{P}_{j+l-1}), \quad \ldots$$

$$\Pi : R^{2\cdot l} \to R^2$$

Dynamics of momenta in the simplest case l=1

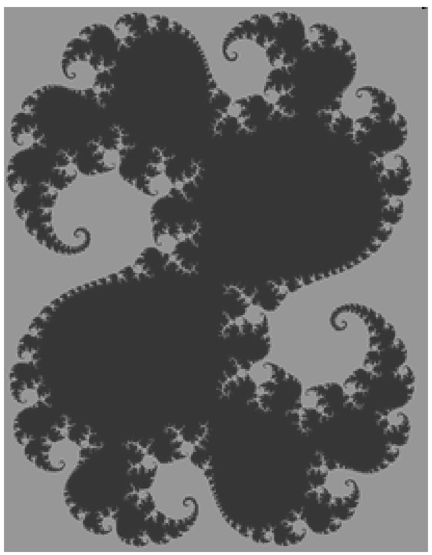
$$P = P_1 + i \cdot P_2$$

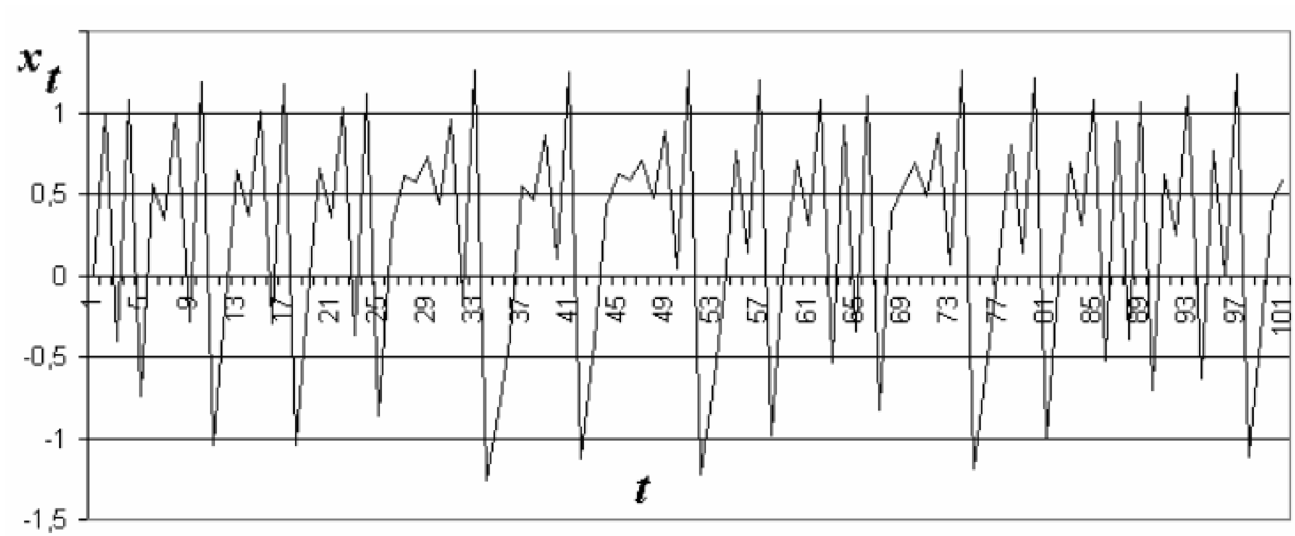
Henon mapping:

$$P_{j+1} = g(P_j)$$

$$P_{1,j+1} = 1 - 1{,}4 \cdot P_{1,j}^2 + P_{2,j} \qquad P_{2,j+1} = 0{,}3 \cdot P_{1,j}$$

Julia set:



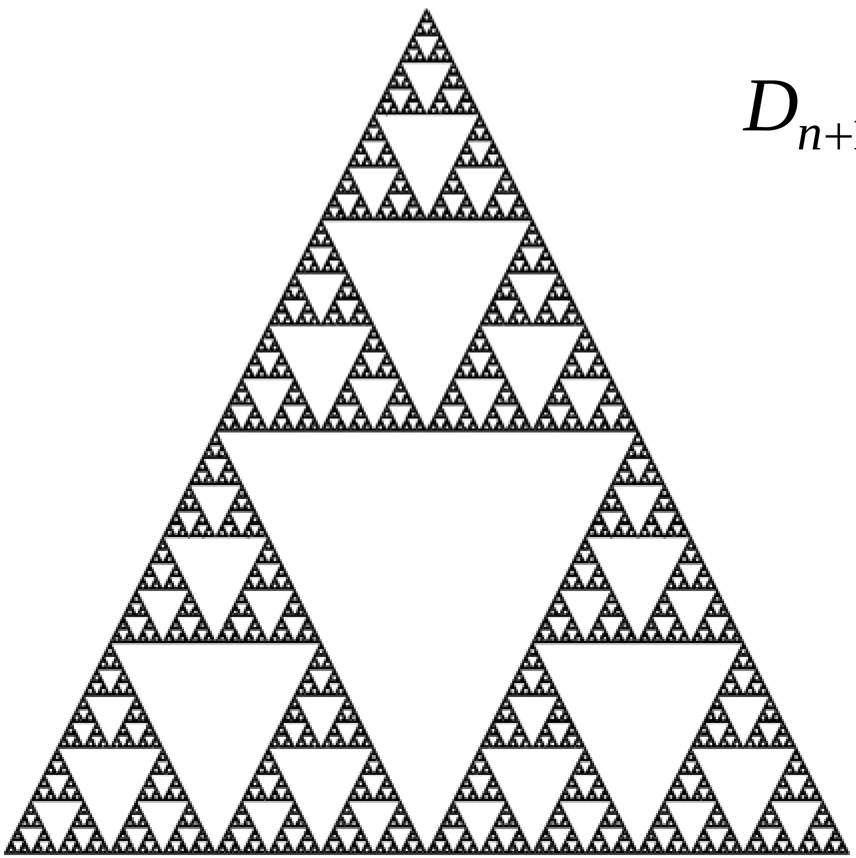


This map have chaotic behaviour!

$$g(P) = P^2 + c$$

The example of construction of fractal set on two-dimensional plane of momenta by means of application of Williams-Hatchinson theorem

$$f_i(\vec{x}) = \frac{1}{2}\cdot\vec{x} + \vec{b}_i \qquad \vec{b}_1 = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \qquad \vec{b}_2 = \frac{1}{2}\cdot\begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad \vec{b}_3 = \frac{1}{4}\cdot\begin{pmatrix} 1 \\ \sqrt{3} \end{pmatrix}$$



$$D_{n+1} = f_1(D_n) \bigcup f_2(D_n) \bigcup f_3(D_n)$$

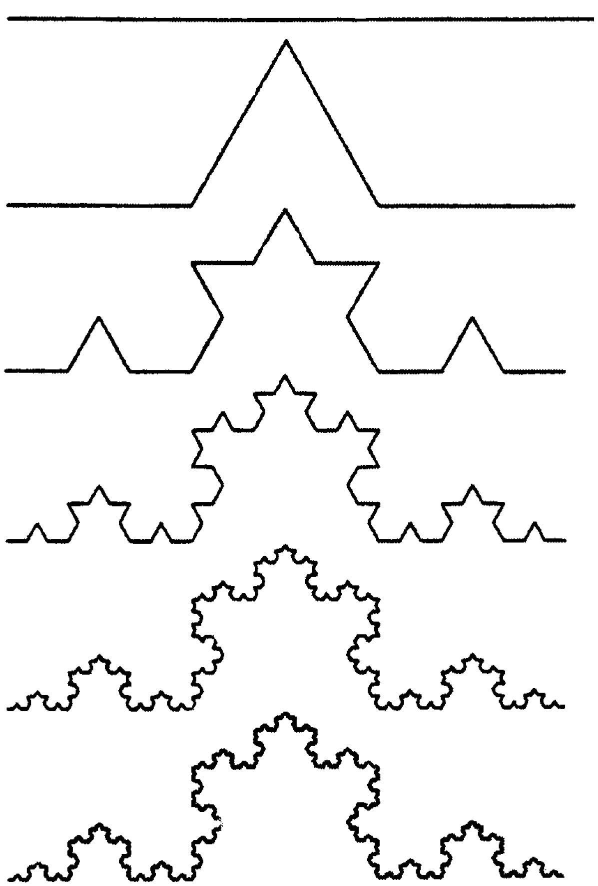
$$\lim_{n\to+\infty} \rho_H(D_n, D) = 0$$

D is Sierpinski napkin. Its fractal dimension is equal to:

$$D_{fr} = \log_2 3 \approx 1{,}58$$

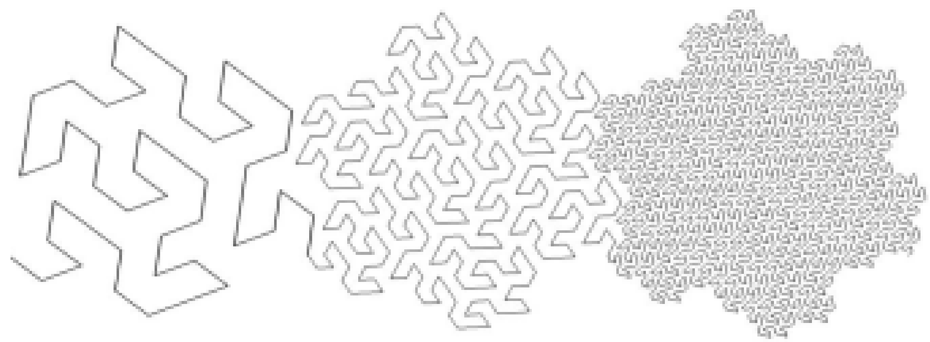
Fractal dynamics on plane of coordinates:

Iterations of Koch's curve

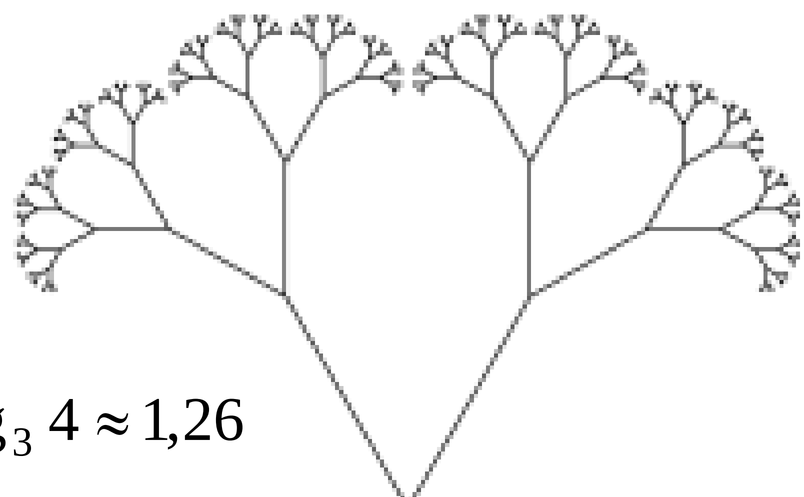


$$D_{fr} = \log_3 4 \approx 1{,}26$$

Iterations of Gosper curve:



Fractal tree:



Nonlocal quantum field theory for surface hydron

$$\frac{\partial^2 \hat{\Phi}(t,\vec{x})}{\partial t^2} + g \cdot \sqrt{-\Delta_{\perp}} \cdot \hat{\Phi}(t,\vec{x}) = 0 \qquad \omega(\vec{p}) = +\sqrt{g \cdot |\,\vec{p}\,|}$$

$$\hat{\Phi}(t,\vec{x}) = \int [\hat{c}(\vec{p}) \cdot \exp(i \cdot \vec{p} \cdot \vec{x} - i \cdot \omega(\vec{p}) \cdot t) + h.c.] \cdot \frac{d^2 p}{\sqrt{2 \cdot \omega(\vec{p})} \cdot 2 \cdot \pi}$$

Bose canonical commutative relations:

$$[\hat{c}(\vec{p}), \hat{c}^{+}(\vec{p}\,')] = \delta(\vec{p} - \vec{p}\,')$$

Fock space for surface hydron:

$$\hat{c}(\vec{p})\,|\,0> = 0$$

$$|\,\vec{p}_1, \vec{p}_2, \ldots, \vec{p}_s >= \hat{c}^{+}(\vec{p}_1) \cdot \hat{c}^{+}(\vec{p}_2) \cdot \ldots \cdot \hat{c}^{+}(\vec{p}_s)\,|\,0>$$

Hamiltonian of surface hydron:

$$\hat{H} = \int : \left[\frac{1}{2} \cdot \left(\frac{\partial \hat{\Phi}(t, \vec{x})}{\partial t} \right)^2 + \frac{g}{2} \cdot \left((-\Delta_{\perp})^{\frac{1}{4}} \cdot \hat{\Phi}(t, \vec{x}) \right)^2 \right] : d^2 x$$

$$\hat{H} = \int \sqrt{g \cdot |\vec{p}|} \cdot \hat{c}^{+}(\vec{p}) \cdot \hat{c}(\vec{p}) \cdot d^2 p$$

$$\hat{H} \,|\, \vec{p}_1, \ldots, \vec{p}_s > = [\omega(\vec{p}_1) + \ldots + \omega(\vec{p}_s)] \cdot |\, \vec{p}_1, \ldots, \vec{p}_s >$$

$$i \cdot \frac{\partial \,|\, \psi >}{\partial t} = \hat{H} \,|\, \psi >$$

— the Schrödinger equation for the wave function of surface hydrons in the Fock space

$$|\, \psi(t) > = \sum_{s=0}^{\infty} \int a_s(\vec{p}_1, \ldots, \vec{p}_s; t) \cdot |\, \vec{p}_1, \ldots, \vec{p}_s > d\vec{p}_1 \cdot \ldots \cdot d\vec{p}_s$$

The results obtained can be easily extended on the following more complicated situations for linear gravitational waves:

$$\omega(\vec{p}) = +\sqrt{g\cdot|\vec{p}| + \frac{\sigma\cdot|\vec{p}|^3}{\rho}}$$

— case of infinitely deep fluid with surface tension

$$\omega(\vec{p}) = +\sqrt{g\cdot|\vec{p}|\cdot th(|\vec{p}|\cdot h)}$$

— case of fluid with finite deep

$$\omega(\vec{p}) = +\sqrt{\left(g\cdot|\vec{p}| + \frac{\sigma\cdot|\vec{p}|^3}{\rho}\right)\cdot th(|\vec{p}|\cdot h)}$$

— case of fluid with finite deep and with surface tension

$$\omega(\vec{p}) = +\sqrt{\frac{g\cdot|\vec{p}|\cdot(\rho_1 - \rho_2)}{\rho_1\cdot cth(|\vec{p}|\cdot h_1) + \rho_2\cdot cth(|\vec{p}|\cdot h_2)}}$$

— case of layers of fluids with different densities

Further developments:

- **investigation of effects of multifractality in trajectories of surface hydron;**
- **investigation of stochastic fractals in trajectories of surface hydron;**
- **calculation of Green's functions by means of Maslov canonical operator;**
- **extension of the approaches developed on elastic and electromagnetic waves in layered media and also for zone theory of solid state;**
- **extension of the methods developed on the case of some branches of dispersion relation.**

References:

- A. A. Potapov, A. E. Rassadin. On Some New Analogies Between Radar-location and Quantum Mechanics // Proc. From XX Int. Scientific-Researh Conf. ‘Radar-location, Navigation, Communication’, Voronezh: NPF “Sakvoee”, 2014, V. 3. pp. 1799-1808.
- A. A. Potapov, A. E. Rassadin. Feynman Integrals, Fractal Radio Systems, or Do We Understand What is Radio Engineering? // Proc From 17th Int. Conf. and Exh. ‘Digital Signal Processing and Its Applications’, Moscow, 2015, V.1, pp. 103-109.
- A. A. Potapov, A. E. Rassadin. On Connection Between Radar-location, Fractal Paradigm and Symmetry of Strong Interactions // Proc. From XXI Int. Scientific-Researh Conf. ‘Radar-location, Navigation, Communication’, Voronezh: NPF “Sakvoee”, 2015, V. 2. pp. 582 - 593.
- A. A. Potapov, A. E. Rassadin . Feynman integrals as connecting component between radio engineering, fractal paradigm and quantum mechanics // Physics of wave processes and radio systems (in print).
- A. A. Potapov, A. E. Rassadin. Fractal Paradigm and New Kind of Quantum Quasiparticles in Hydroacoustics // Proc. From IX Int. Scientific Conf. ‘Chaos and Structures in Nonlinear Systems. Theory and Experiment’, Karaganda: E.A. Buketov KarSU Publishing House, 2015, pp. 536-541.
- A. A. Potapov, A. E. Rassadin. Feynman integrals, fractal paradigm and new point of view on hydroacoustics // Eurasian Physical Technical journal (in print).
- A. A. Potapov, A. E. Rassadin. On new properties of quasiparticle of radiosystems // Proc. From 25th Int. Crimean Conf. ‘Microwave & Telecommunication Technology’ (in print).
- A. A. Potapov, Yu. V. Gulyaev, S. A. Nikitov and alt. The modern methods of image processing. 2008, M., Fizmatlit, 496 p.
- S. A. Podosjonov, A. A. Potapov, J. Foukzon and alt. Nonholonomic, fractal and connected structures in relativistic continuous media, electrodynamics, quantum mechanics and cosmology / Under ed. of A. A. Potapov. In 3 v. 2015, M., LENAND, 1200 p.
- Potapov A. A., Rassadin A. E. Fractals and path integrals in three-dimensional wave equation // Book of abstracts of Int. Conference-School “Infinite-dimensional dynamics, dissipative systems, and attractors” (Nizhny Novgorod, Russia, July 13-17, 2015) p. 26-27.
- Potapov A. A., Rassadin A. E. Feynmanons in the Korteweg-de Vries equation // Book of abstracts of Int. Conference-School “Infinite-dimensional dynamics, dissipative systems, and attractors” (Nizhny Novgorod, Russia, July 13-17, 2015) p. 28-29.