Fermi acceleration in billiards with holes

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Work done jointly with :
Vassili Gelfreich, Vered Rom-Kedar and Dmitry Turaev

Introduction to Fermi Acceleration

- Particle moving within a dynamical billiard with moving walls
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- Particle moving within a dynamical billiard with moving walls
  - How fast does the energy of a particle ensemble grow?
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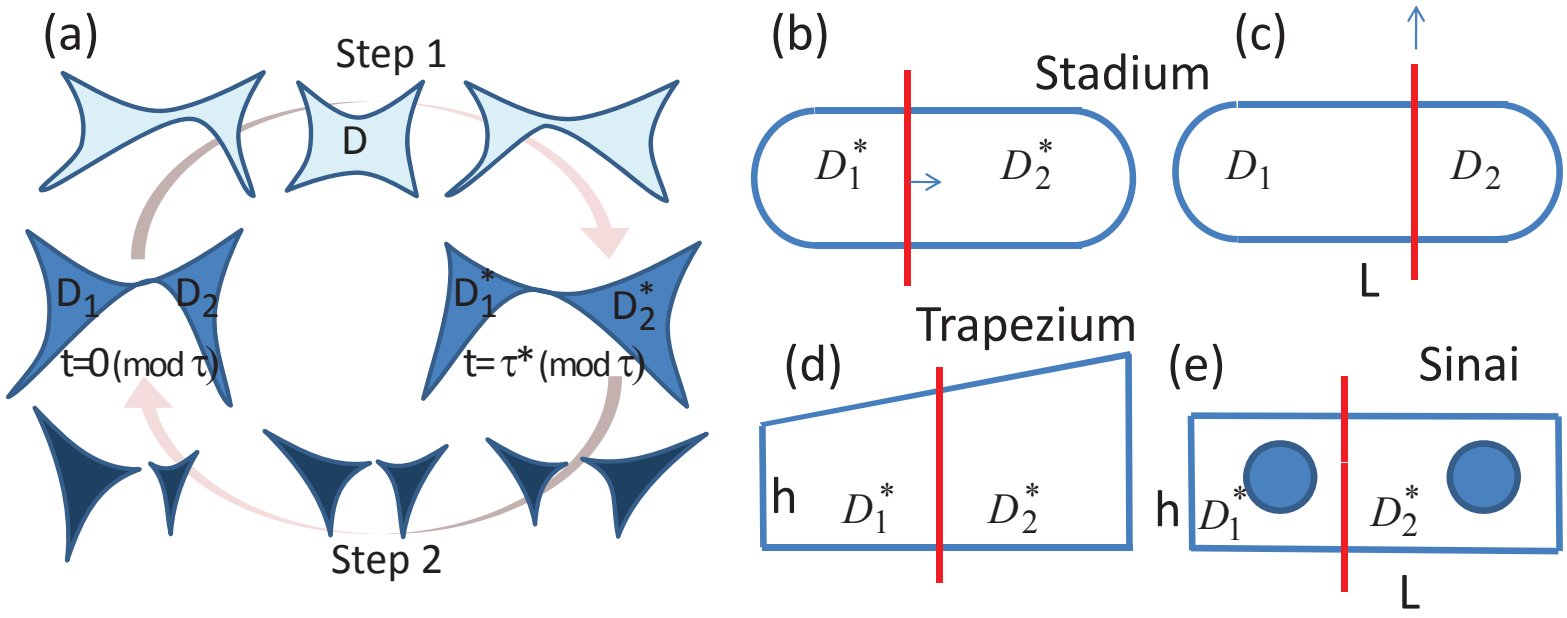
  - Billiard with mixed phase-space : Exponential energy growth

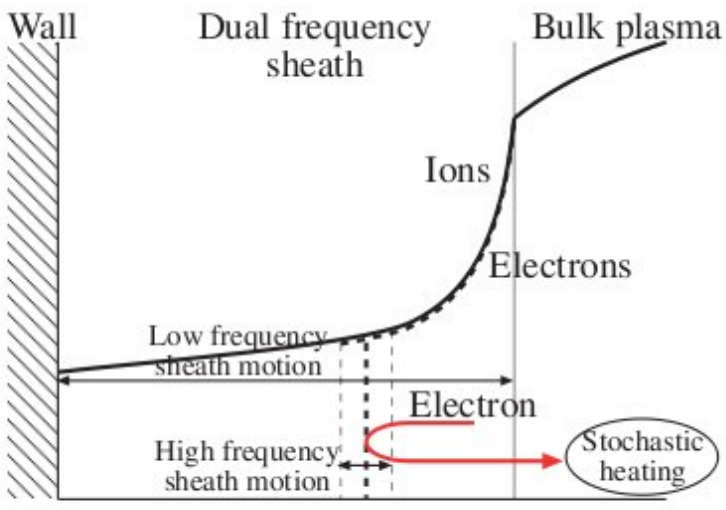
Robust exponential accelerator



V. Gelfreich, V. Rom-Kedar, K. Shah and D. Turaev, Phys. Rev. Lett. 2011

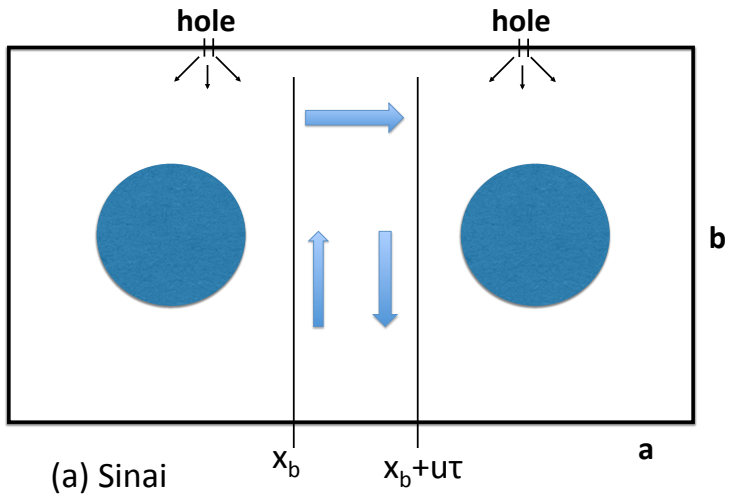
Applications of Fermi acceleration

- Origin of cosmic rays
- Heating of charged particles (plasma) in RF traps

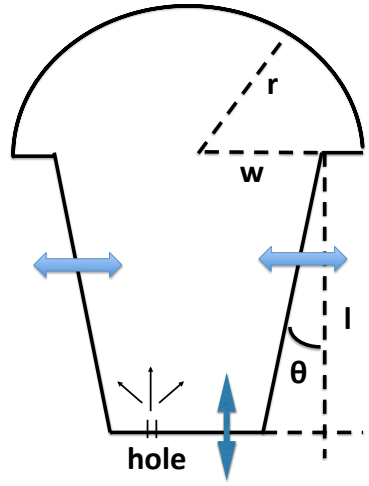


- Motion of ultracold atoms in dark optical traps
  - Study transition from classical to quantum dynamics
- Nuclear Fission

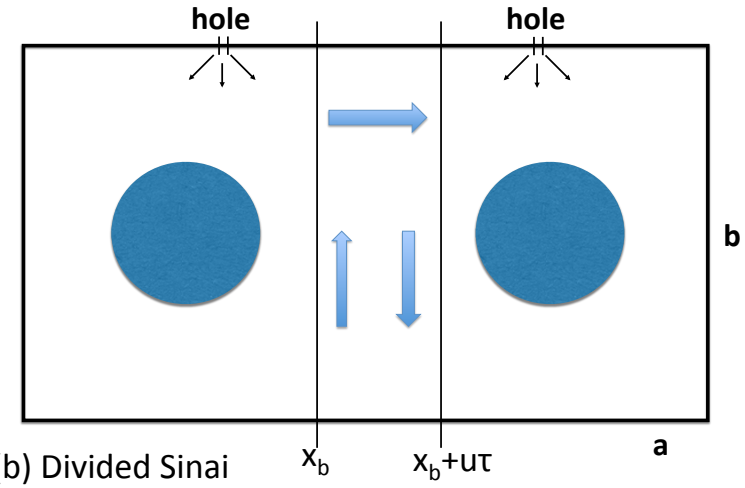
Billiards with holes



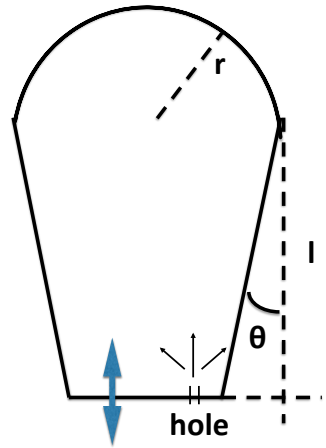
(a) Sinai



(c) Mushroom



(b) Divided Sinai



(d) Stadium

Leaky Fermi Accelerator : Math Model

$$\text{Energy gain : } G\left(E_{in}\right) = h\nu_{in}\left[E_{out} - E_{in}\right]$$

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Assumptions

- Hole size, h, is small and does not effect the billiard statistics
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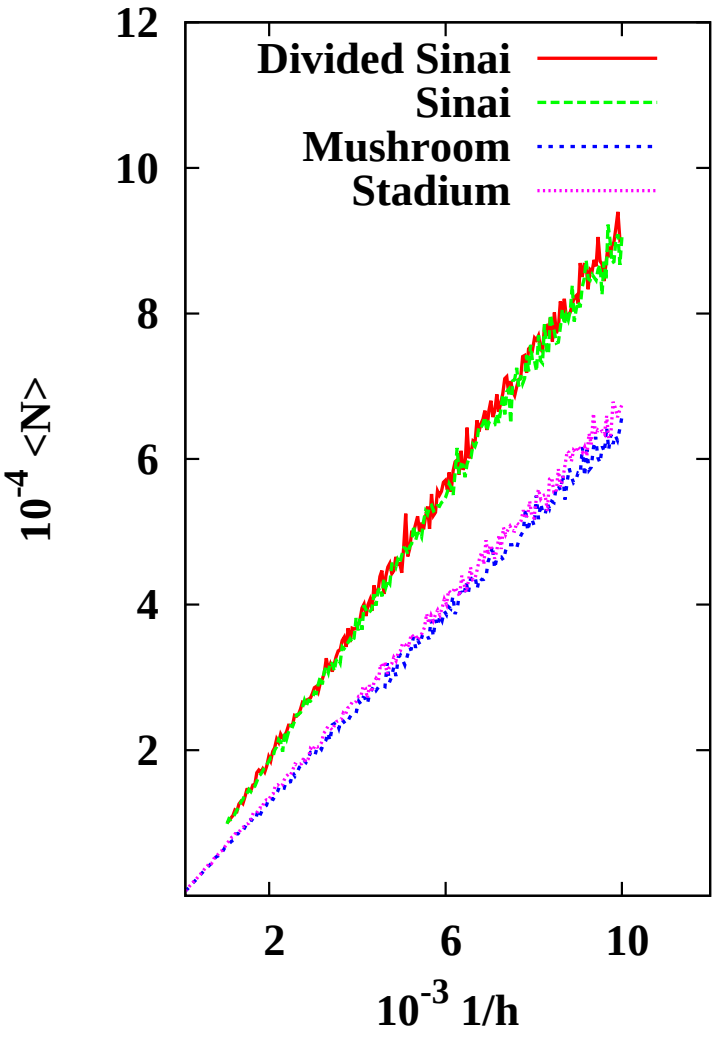
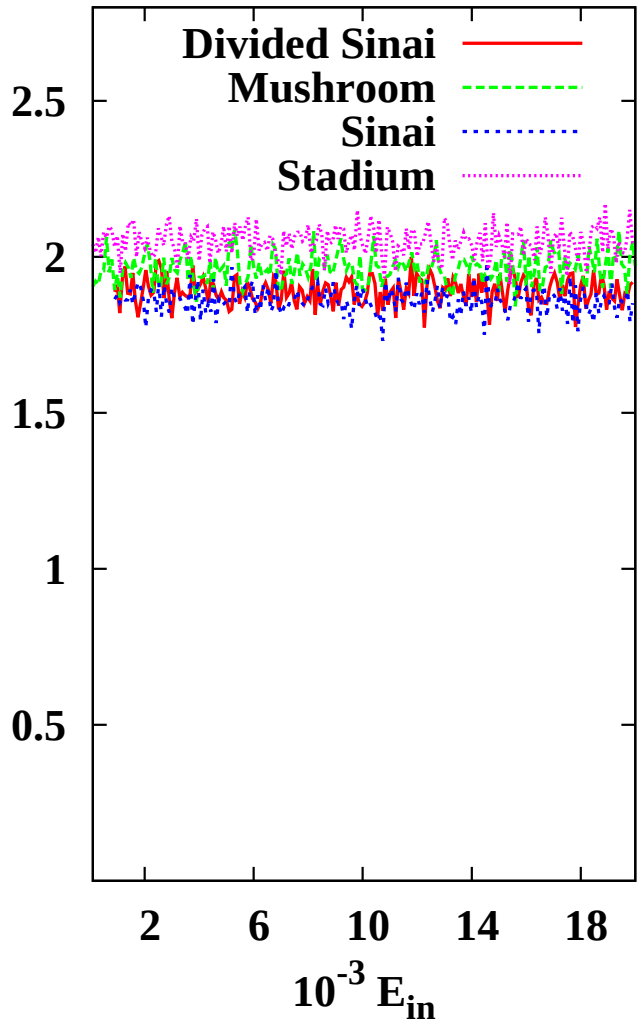
- Wall velocity is much less than particle velocity, $|u| \ll v$

Average number of collisions





Mathematical Predictions

Ergodic Case :

Non-ergodic Case :

Mathematical Predictions

Ergodic Case :

$$\begin{aligned} E_{out} - E_{in} &= k\frac{\overline{u}^2}{2}\langle N\rangle \\ \Rightarrow G\left(E_{in}\right) &= hv_{in}\left[E_{out} - E_{in}\right] \end{aligned}$$

Non-ergodic Case :

Mathematical Predictions

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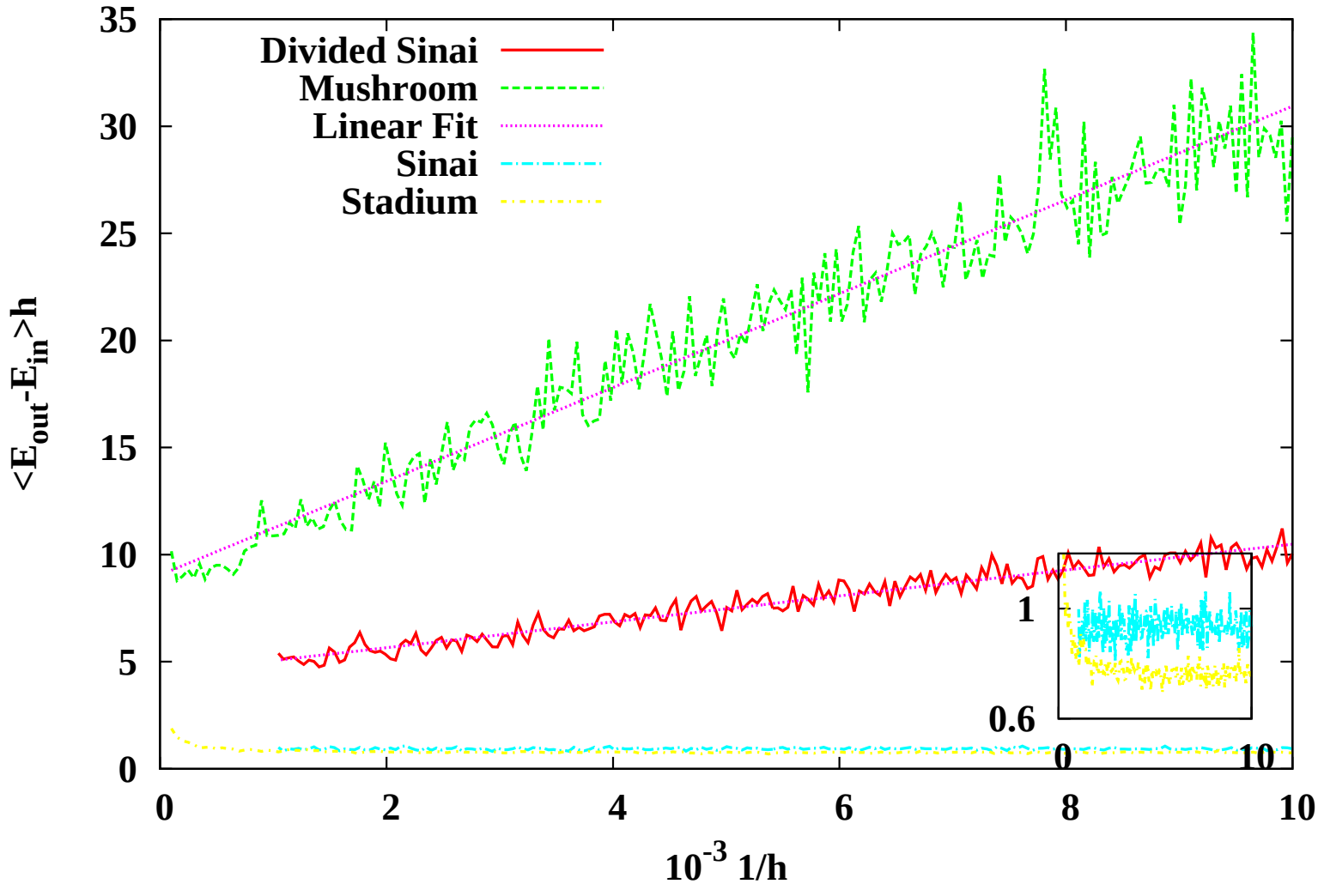
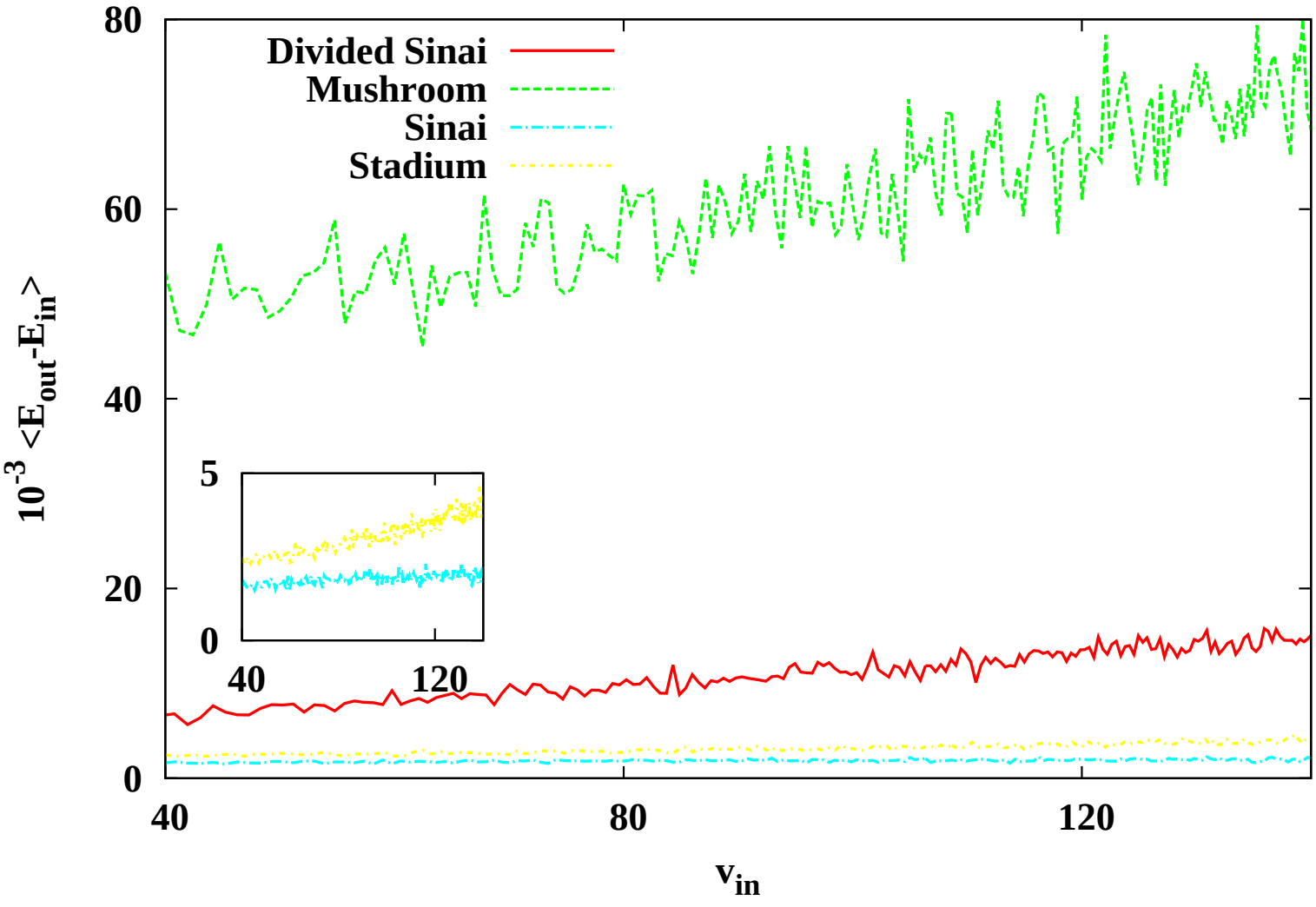
$$\begin{aligned} E_{out} - E_{in} &= \frac{\overline{u}^2}{2}\left[k_1\frac{v_{in}T}{L}\langle N\rangle + k_2\left(\frac{\overline{u}T}{L}\right)^2\langle N^2\rangle\right] \\ G\left(E_{in}\right) &= v_{in}\frac{V}{L}\frac{\overline{u}^2}{2}\left[k_1\frac{v_{in}T}{L} + k_2\left(\frac{\overline{u}T}{L}\right)^2\frac{V}{Lh}\right] \end{aligned}$$

Simulation : Energy gain vs. Hole size



Simulation : Energy gain vs. Initial velocity



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Physical Review E 91, 062920 (2015)

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  - Collisions may impede heat production for connected billiard

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